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On some generalized recent operators in topological Spaces

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ABSTRACT

In this paper we introduce a new classes of operators namely (Λ regular generalized*, Λ regular generalized**, Λ generalized **regular) (briefly, Λrg*,Λ rg **, Λ g**r, respectively) in topological spaces and study some of their properties with some important theorems.

1. Introduction

In 1971 [1] defined semi-closure in topological space. In (1982) [2] studied the concept of closure operator. In (2003) [6] used the concepts of open sets, closure operator, and semi-closure to present and analyze several weak separation axioms. Later, [7] used the notions of pre-open sets and pre-closure operator to develop certain weak separation axioms. Some theorems of this nature theorem have been proved by some researchers: Let (X,T) be a T.S. (topological space) and A subset of X, is the intersection of all closed (resp. semi-closed, pre-closed, semi-pre-closed, b-closed) sets of X containing A is called the closure of A the closure of A [5](resp. semi-closure [1], pre-closure [3], semi-pre-closure [4], b-closure [5]) of A

In (2018) [8] introduced πg*β-closure in a topology space. In the year (2020) [9] and [10] introduced Pre-weakly generalized closed sets and presented Soft –interior and soft –closure in soft topology .

In this paper we present a new class of different operators we knew him on regular generalized open sets namely (Λ regular generalized*, Λ regular generalized**, Λ generalized** regular) (briefly, Λrg*, Λ rg**, Λ g **r, respectively) and study some of their properties with some important theorems.

2. Preliminaries

This section of the manual lists some of the required definitions and theorems.

Definition 2.1[11]: A subset A of a space X is called regular generalized*- open [resp. regular generalized**- open] set (briefly rg*-o(x), rg**-o(x)) if int(A) ⊆U , whenever U⊆A and U is regular [resp. pre regular] closed set ⊆ X.

Definition 2. 2[11]: A subset A of a space X is called generalized** regular open set (briefly, g**r-o(x)) if Rint (A) ⊆U , whenever U⊆A and U is pre-regular closed set ⊆X.

Definition 2.4[12]: Let (X, τ) is a topological space as well as the operator Λ_{GR} (F)= ∩{U:F ⊆U, U is generalized regular open set ⊆ X}

3. On Some generalized recent operators in topological spaces

Definition 3.1: The new operators are listed below. Allow K to take the initiative = [rg*,rg**] sets . Λ_K(A)=∩{U: A ⊆U , U is K∈ (rg*,rg**) open ⊆ X}

Example 3.2: X={a, b, c, d, e} , τ ={∅ ,X,{a, c, e},{a, b ,c ,e},{a ,b}}

τ^c={∅ ,X,{d},{b, d},{c ,d, e}}

R-o(x)={ ∅ ,X}, R-c(x)={X, ∅}.

PreR-o(x)={∅,X,{a},{b},{c},{d},{e},{a,b},{a,c},{a,e},{a, d},{b,c},{b,d},

{b,e},{c,e},{c,d},{a,b,c},{a,b,d},{a,b,e},{a,c,e},{a,c,d},{b,c,d},{b,c,e},{c,d,e}
,{b,d,e},{a,d,e},{a,b,c,d},{a,b,c,e},{a,c,d,e},{b,c,d,e},{a,b,d,e}}

PreRc(x)={∅,X,{a},{b},{c},{d},{e},{a,b},{a,c},{a,e},{a,d},{b,c},{b,d},{b,e},{c,e},{c,d},{d,e},{a,b,d},{a,b,e},{a,c,e},{a,c,d},{b,c,d},{b,c,e},{c,d,e},{b,d,e},{a,d,e},{a,b,c,d},{a,b,c,e},{a,c,d,e},{b,c,d,e},{a,b,d,e}}

A={a ,b ,c} , ΛK (A)=X ,

K=(rg*, rg**)-open sets={∅,X{a},{d},{e},{b, c},{b, d},{c ,e},{d ,e},
{a, c ,d},{b ,c ,d},{a ,d ,e},{c, d, e},
{b ,c ,d, e}}

Lemma 3.3: (X, τ) be a topological space, Λ_K: q(x) → q(x) is operator which satisfies the properties.

1. $\Lambda_K(\emptyset)=\emptyset$, $\Lambda_K(X)= X$
2. . $\Lambda_K(A)\supseteq A$
3. $\Lambda_K\Lambda_K(A)= \Lambda_K(A)$
4. If B⊇A, $\Lambda_K(B)\supseteq \Lambda_K(A)$.
5. $\Lambda_K(A \cap B)\subseteq \Lambda_K(A) \cap \Lambda_K(B)$.
6. $\Lambda_K(A \cup B)\supseteq \Lambda_K(A) \cup \Lambda_K(B)$.

Proof: 2- $\Lambda_K(A)\supseteq A$, Since A ⊆ K , K∈ (rg*,rg**)open sets ⊆X , by definition (3.1) we get A⊆ $\Lambda_K(A)$

3- $\Lambda_K\Lambda_K(A)= \Lambda_K(A)$, Λ_K (A)=∩{G:A ⊆G, G is K∈ (rg*,rg**)open ⊆X } .

$\Lambda_K[\Lambda_K (A)]=$ ∩ [∩{G:A ⊆G, G is K∈ (rg*,rg**)open ⊆X }]
=∩{G:A ⊆G, G is K∈ (rg*,rg**)open ⊆X }= $\Lambda_K(A)$

4- If B⊇A, $\Lambda_K(B)\supseteq \Lambda_K(A)$, Λ_K (A)=∩{G:A ⊆G, G is K∈ (rg*,rg**)open ⊆X }.

Since A⊆ S, then
∩{G:D ⊆G, G is K∈ (rg*,rg**)open ⊆X }⊆ ∩{G:S ⊆G, G is K∈ (rg*,rg**)open ⊆X }⟹ $\Lambda_K(B)\supseteq \Lambda_K(A)$.

5- $\Lambda_K(A \cap B) \subseteq \Lambda_K(A) \cap \Lambda_K(B)$, since A ∩ B ⊆ A and A ∩ B ⊆ B

then $\Lambda_K(A \cap B)\subseteq \Lambda_K(A)$ and $\Lambda_K(A \cap B)\subseteq \Lambda_K(B)$.

Hence $\Lambda_K(A \cap B)\subseteq \Lambda_K(A) \cap \Lambda_K(B)$.

6- $\Lambda_K(A \cup B)\supseteq \Lambda_K(A) \cup \Lambda_K(B)$.

Since A ⊆ A ∪ B and B ⊆ A ∪ B

⟹$\Lambda_K(A) \subseteq \Lambda_K(A \cup B)$, $\Lambda_K(A) \subseteq \Lambda_K(A \cup B)$.

Hence $\Lambda_K(A \cup B)\supseteq \Lambda_K(A) \cup \Lambda_K(B)$.

Remark 3.5: The following example shows that the converse section (5-6) of lemma (3.3) is not valid.

Example3.6: X={a ,b ,c, d ,e}

τ ={∅ ,X,{e},{a ,b},{d ,e},{a ,b, e},{c, d ,e},{a ,b ,c ,d},{a ,b ,d ,e}}, Pro(x)={∅ ,X ,p(x)}} ,U={a, b, c, d ,e}

(rg*,rg**)open sets={∅,X,{a},{ b},{c},{a, d},{{b ,d},{b, c},{c ,d},{a ,c ,d},{b ,c ,d}}.

i. A={b}, B={a ,c ,d}, $\Lambda_K(A)$={b ,d}, $\Lambda_K(B)$={a, c, d}, $\Lambda_K(A \cup B)$=X , but $\Lambda_K(A) \cup \Lambda_K(B)$={a ,b ,c ,d}.then $\Lambda_K(A \cup B) \neq \Lambda_K(A) \cup \Lambda_K(B)$.

ii. A={c, d ,e}, B={a ,b ,c}, $\Lambda_K(A)$=X, $\Lambda_K(B)$=X, $\Lambda_K(A \cap B)$={c}
$\Lambda_K(A) \cap \Lambda_K(B)$=X. , then $\Lambda_K(A \cap B) \neq \Lambda_K(A) \cap \Lambda_K(B)$

Lemma 3.7: (X, τ) be a topological space, $\Lambda_{g^{**}r}$: L(x) ⟶ L(x) is operator satisfy the conditions.

1- $\Lambda_{(g^{**}r)}(\emptyset)=\emptyset$, $\Lambda_{(g^{**}r)}(x)=x$

2-$\Lambda_{(g^{**}r)}(S)\supseteq S$

3-$\Lambda_{(g^{**}r)} \Lambda_{(g^{**}r)}(A)= \Lambda_{(g^{**}r)}(A)$

4-If B⊇A, $\Lambda_{(g^{**}r)}(B)\supseteq \Lambda_{(g^{**}r)}(A)$

5-$\Lambda_{(g^{**}r)}(A \cap B)\subseteq \Lambda_{(g^{**}r)}(A) \cap \Lambda_{(g^{**}r)}(B)$

6-$\Lambda_{(g^{**}r)}(A \cup B)\supseteq \Lambda_{(g^{**}r)}(A) \cup \Lambda_{(g^{**}r)}(B)$.

Proof**:** (2- 3-4-5-6) The same steps as the previous proof of lemma(3-3) .

Remark 3.8: The following example shows that the converse section (5-6) of lemma (3.7) is not valid.

Example 3.9: Let X={a, b ,c, d ,e} and the corresponding topology be

τ ={∅ ,X,{d},{ c, e},{c ,d ,e},{a ,b ,c, e}}

τ^c={∅ ,X,{d},{a ,b},{a ,b, d},{a ,b, c ,e}}

R-o(x)={ ∅ ,X,{d},{a ,b, c ,e}}, R-c(x)={X, ∅,{a ,b, c ,e },{d }},.

PreRo(x)={∅,X,{a},{b},{c},{d},{e},{a,b},{a,c},{a,e},{a,d},{a,e},{b,c},{b,d},{b,e},{c,e},{c,d},{d,e},{a,b,c},{a,b,d},{a,b,e},{a,c,e},{a,c,d},{b,c,d},{b,c,e},{c,d,e},{b,d,e},{a,d,e},{a,b,c,d},{a,b,c,e},{a,c,d,e},{b,c,d,e},{a,b,d,e}}

PreRc(x)={∅,X,{a},{b},{c},{d},{e},{a,b},{a,c},{a,e},{a,d},{a,e},{b,c},{b,d},{b,e},{c,e},{c,d},{d,e},{a,b,c},{a,b,d},{a,b,e},{a,c,e},{a,c,d},{b,c,d},{b,c,e},{c,d,e},{b,d,e},{a,d,e},{a,b,c,d},{a,b,c,e},{a,c,d,e},{b,c,d,e},{a,b,d,e}}

g**r-open set={∅,X,{a},{b},{c},{e},{a,b},{a,c},{a,e},{a,e},{b,c},{b,e}
,{c,e},{a,b,c},{a,b,e},{a ,c, e},{b ,c ,e}}.

1- A={e}, B={a ,c ,d}, $\Lambda_K(A)$={e}, $\Lambda_K(B)$={a, c, d}, $\Lambda_K(A \cup B)$= X ,but $\Lambda_K(A) \cup \Lambda_K(B)$={ ,{a, c, d, e} , then $\Lambda_K(A \cup B) \neq \Lambda_K(A) \cup \Lambda_K(B)$.

2- A={c, d ,e}, B={a ,b ,d}, $\Lambda_K(A)$=X, $\Lambda_K(B)$=X, $\Lambda_K(A \cap B)$={d}
$\Lambda_K(A) \cap \Lambda_K(B)$=X , then $\Lambda_K(A \cap B) \neq \Lambda_K(A) \cap \Lambda_K(B)$

Theorem 3.10: $\Lambda_{(rg^{*},rg^{**})}(A)=A$ ↔ A is a regular generalized* [resp. regular generalized**] open set.

Proof: Let A be a regular generalized* open [resp. regular generalized** open] set.

Since A ⊆ $\Lambda_{rg^{*},rg^{**}}(A)$ and int(A) ⊆ U, U is regular [resp. pre regular] closed set then
$\Lambda_{rg^{*},rg^{**}}(A) \subseteq$ A , therefore $\Lambda_{rg^{*},rg^{**}}(A)$= A .

Conversely: if $\Lambda_{rg^{*},rg^{**}}(A)$= A . To prove A is a regular generalized* [resp. regular generalized**] open set

Since arbitrary intersection of regular generalized* [resp. regular generalized**] open set is regular generalized* [resp. regular generalized**] open set. Then A is a regular generalized* [resp. regular generalized**] open set.

Theorem 3.11: $\Lambda_{(g^{**}r)}(A)$= A ↔ A is generalized regular** open set.

Proof: Let A be a generalized regular** open set.

Since A ⊆ $\Lambda_{gr^{**}}(A)$, Rint(A) ⊆ U, U is Pre regular closed set. then $\Lambda_{gr^{**}}(A) \subseteq A$, , therefore $\Lambda_{gr^{**}}(A)$= A .

Conversely: if $\Lambda_{gr^{**}}(A)= A$. To prove A is a generalized regular** open set

Since arbitrary intersection of generalized regular** open set is generalized regular** open set

Hence A is generalized regular** open set.

4. Conclusions

We introduced a new type of operators namely (Λ regular generalized*, Λ regular generalized**, Λ generalized **regular) (briefly, Λrg*,Λ rg **, Λ g**r, respectively) in topological spaces and study some of their properties with some important theorems. I n the next study we can generalized to the Nano topological space and micro topological space. As well as the definition of operator Λ can generalized on (α, β and b) open sets.

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على بعض العوامل المعممة الحديثة في الفضاءات التبولوجية

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الملخص

في هذا البحث نقدم فئات جديدة لبعض العوامل اسميناها (Λ regular generalized, Λ regular generalized, Λ generalized regular). (باختصار ، * Λ rg ، ** Λ rg ، Λ g** r ، على التوالي) في الفضاء التبولوجي وأعطينا بعض الأمثلة الهامة ودرسنا بعض خصائصها وبعض النظريات .