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Nuclear energy levels in ^{58}Ni

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ABSTRACT

Energy levels, total angular momenta and parity for two nucleons that present outside closed core ^{56}Ni (Nickel) which occupied FP nuclear shell have been calculated using nuclear shell model application. We have used FP M3YE interaction to calculate energy levels of ^{58}Ni, in addition, the results are compared with experimental data, modified surface delta interaction (MSDI), Reid's potential (RP) and non-zero pairing shell model (NZPSM).

Harmonic Oscillator potential is used to generate single particle wave functions in FP shell and considering $^{56}_{28}Ni_{28}$ as an inert core.

Oxford Buenos Aires Shell Model (OXBASH) code for nuclear shell model calculation has been utilized to carry out the calculations and comparison with experimental data had been done.

1. Introduction

$F_{7/2}$, $P_{3/2}$ and $P_{1/2}$ (FP Shell) are well used model space and Michegan some of three range Yukawa potential (M3Y) is the most suitable effective interaction to create nuclear energy levels in ^{58}Ni.

Many different microscopic theories performed to study nuclei excitations. The multi-nucleon shell model with mixed configurations is one of these theoretical models. In this model, the nucleus ^{56}Ni target is assumed as an inert core with addition effective nucleons distributed over a limited number of orbits, which called space-orbit model. Shell-model calculations carried out within a model-space in which the nucleons are restricted to occupy a few orbits are unable to reproduce the measured static moments or transition strengths without scaling factors [1].

The shell model is an essential section of nuclear theory and a basic theoretical tool for the microscopic description of nuclear structure. The fundamental assumption in the nuclear shell model is that each nucleon moves independently in an average field includes a strong spin-orbit term and produced by the nucleons itself. The nucleons then arrange themselves into group of levels, the "shells", well separated from each other. According to this approximation, the nucleus is considered to be consisting of an inert core made up by shells filled up (closed shell) with neutrinos and protons plus a certain number of external nucleons called valence nucleons [2].

Experimental data must be present in order to make sure that the carried theoretical calculations meet the experimental result with an acceptable errors or differences and the experimental data are reliably enough with respect to the circumstances that's affecting the calculation and comparison. Results must be clear and carefully viewed in order to reveals the details [1].

Several review articles have been published discussing the development of this topic, which one can find actuated summary of what has been done in this field.

In 1964, Y. Waghmare et al had analyzed the energy level spectrum of ^{58}Ni on the simple considerations of pure singlet s-state forces [3]. In 1974, S.P. Pandia and B.P. Singh had calculated the structure of the low-lying of ^{58}Ni in shell model by assuming an inert ^{56}Ni core plus two valence nucleons in the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ orbitals [4]. In 2002, S. Tagesen et al had performed a new evaluation of all important neutron cross sections of ^{58}Ni in the neutron energy range (0.812-20) MeV[5]. And In 2007, Y. Kalmykov et al have extracted Level densities of $J^{\pi}=2^{+}$ and 2^{-} states from high-resolution studies of E2 and M2 giant resonances in ^{58}Ni and ^{90}Zr [6].

The aim of this work is to reproduce nuclear energy levels in ^{58}Ni, by using realistic interactions and use of modern version of fitting parameter sets.

2. Theory

2.1. The nucleus ^{58}Ni

Nickel (Ni) possesses five stable isotopes including ^{58}Ni, ^{60}Ni, ^{61}Ni, ^{62}Ni and ^{64}Ni. In addition, 27 radioactive isotopes have been discovered ranging from ^{48}Ni to ^{79}Ni, some of them have short half-lives. The longest half -lives is ^{59}Ni with a half-life of 7.6×10^4 years. Most of them are less than a minute or a second. The least unstable is ^{79}Ni with a half-life of $635 \times 10^{-9}\ s$ [1].

The nucleus ^{58}Ni have 28 protons and 30 neutrons, two neutrons play essential role in the model space shell, outer the closed shell when the inert core ^{56}Ni is under consideration.

2.2. Perturbation theory

For the application of perturbation theory it is advantageous to make the influence of the residual interaction [7]:

$$H^{(1)} = \sum_{1=k<l}^{A} W(k,l) - \sum_{k=1}^{A} U(k) \ldots (1)$$

As small as possible. Where $W(k,l)$ is the nucleon-nucleon interaction, and $U(k)$ is a single particle potential.

To be noticed that any product of a single-particle functions $\Phi_a^{(0)} \equiv \emptyset_{a1}(r(1)) \ldots . \emptyset_{aA}(r(A))$ satisfies the Eigen value equation [8]:

$H^{(0)}\Phi_a^{(0)} = E_a^{(0)}\Phi_a^{(0)}$ ...(2)

The first-order perturbation theory is [7]:

$|\Phi_\Gamma\rangle = |\Phi_\Gamma^{(0)}\rangle + |\Phi_\Gamma^{(1)}\rangle$, ...(3)

$|E_\Gamma\rangle = |E_\Gamma^{(0)}\rangle + |E_\Gamma^{(1)}\rangle$, ...(4)

Where $|\Phi_\Gamma^{(1)}\rangle$ and $|E_\Gamma^{(1)}\rangle$ represent the supposedly small changes in the wave function and the energy of the unperturbed state.

2.3. Binding energies and excitation energies

The binding energy $E^b$of a nucleus can be defined as the negative value of the total energy needed to decompose the nucleus into free protons and neutrons. Often the binding energy is given with opposite, positive sign. With the negative sign used here, however, there is a more direct connection with the expectation value of the Hamiltonian of the nuclear system. The absolute value of the binding energy is the largest for the nucleus in its ground state. The excitation energy $E_x(n)$of the nth excited state follows from the binding energy $E^b(n)$ of the nucleus in that state taken with respect to the ground-state binding energy$E^b(0)$ [8]:

$E_x(n) = E^b(n) - E^b(0)$. ...(5)

A worthwhile description of a nucleus can be made in terms of an inert core of closed shells and two extra nucleons in the orbit ρ not occupied by core nucleons (as shown in fig. 1). The various terms contributing to the total binding energy of this nucleus can be written down at once from the definition of the binding energy given above as [9]:

$E_\Gamma^b(core + \rho^2) = 2e_\rho + E_\Gamma^{(1)}(\rho^2) + E^b(core)$, ...(6)

Here the term $2e_\rho$ represents the negative value of the energy needed to remove the two particles from the potential well in which they are assumed to move independently in the orbit ρ. It is usually assumed that this potential well does not depend on the number of particles outside the core. The contribution to the binding energy from the mutual nuclear interaction of the two outer-core particles is given by$E_\Gamma^{(1)}(\rho^2)$. This term depend not only on the orbit ρ, but also on the spin J_Γ and isospin T_Γ of the two particle system. The last term, E^b (core), represents the binding energy of the particles in the core [10].

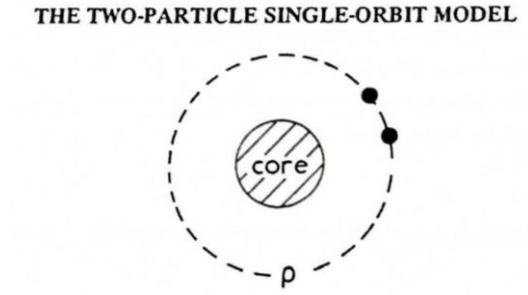

Fig. (1) Schematic illustration of a nucleus described by an inert core and two particles in the orbit ρ [8].

Where it is assumed that the closed-shell core is inert (meaning it will maintain its closed-shell configuration and thus cannot be excited), it follows that the term E^b (core) is a constant [7]. In the case of two active particles outside a core one has [7]:

$H_{12}^{(1)} = V(1,2)$. ...(7)

for the total Hamiltonian [8]:

$H = H_{core} + H_{s.p.}(1) + H_{s.p.}(2) + V(1,2) \ldots (8)$

The binding energy of the nucleus with two particles outside the core in the orbit ρ and coupled to spin and isospin Γ is given by the energy expectation value [11]:

$E_\Gamma^b(A) = \left\langle \Phi_\Gamma^{(0)}(1,\ldots,A) \middle| H \middle| \Phi_\Gamma^{(0)}(1,\ldots,A) \right\rangle \ldots (9)$

Of the total Hamiltonian in the state $\Phi_\Gamma^{(0)}(1,\ldots,A)$ of the complete nucleus.

2.4. States of mixed configuration

The discussion have been restricted to unperturbed states $\Phi_\Gamma^{(0)}$, where Each of these states represents a pure shell-model configuration. For example, considering two active orbits ρ and λ, one has pure states $\Phi_\Gamma^{(0)}(\rho_\alpha^n \lambda_\beta^m)$ each with a given particle distribution specified by ρ^n and λ^m and a given set of intermediate quantum numbers α and β with $J_\alpha + J_\beta = J_\Gamma$ and $T\alpha + T_\beta = T_\Gamma$ [7].

2.5. Energy Matrix Element with Oscillator Function

The Hamiltonian describing the independent motion of two particles in a harmonic-oscillator potential can be separated in (r_1, r_2) space and also in the relative and center of mass coordinate system of the two particles [12]:

$\hat{H} = \frac{p_1^2}{2m} + \frac{1}{2}m\omega^2 r_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2 r_2^2 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2 + \frac{p^2}{2m} + \frac{1}{2}m\omega^2 R^2$. …(10)

t may be shown that this transformation is independent of the magnetic quantum number μ. The transformation bracket vanishes for all combinations of its parameters which do not satisfy the total angular momentum [13]:

$\vec{\lambda} = \vec{l}_1 + \vec{l}_2 = \vec{l} + \vec{L}$ ..(11)

From angular momentum conservation law.

And the degenerated Eigen value (energy) in this case is [13]:

$E(n_1 l_1, n_2 l_2) = \left(2n_1 + l_1 + \frac{3}{2}\right)\hbar\omega + \left(2n_2 + l_2 + \frac{3}{2}\right)\hbar\omega$ … (12)

The wave function $\phi_{nl} = (\vec{r})$ describes states with energy $\left(2n + l + \frac{3}{2}\right)\hbar\omega$ has the form [13]:

$\phi_{nl}(\vec{r}) = R_{nl}(\alpha r)Y_{lm}(\theta, \varphi)$ …. (13)

Where $R_{nl}(\alpha r)$ is the radial wave function and given by [13]:

$R_{nl}(\alpha r) = \left[\frac{2^{l-n+2}(2l+2n+1)!!\alpha^{21+3}}{\sqrt{\pi}n![(2l+1)!!]^2}\right]^{\frac{1}{2}}\left[exp\left(-\frac{\alpha^2}{2}r^2\right)\right]r^l \times \sum_{k=0}^{n}\frac{(-1)^k 2^k n!(2l+1)!!(\alpha^2 r^2)^k}{k!(n-k)!(2l+2k+1)!!}$ …. (14)

and $Y_{lm}(\theta, \varphi)$ is the angular wave function and $\alpha^2 = \frac{m\omega}{\hbar}$.

2.6. The Realistic M3Y Effective Nucleon-Nucleon Interaction:

The realistic M3Y effective NN interaction, which is used in energy level calculations (V_{res}) is expressed as a sum of V_C the central potential part, $V_{S.l}$ spin-orbit potential part, and V_{ten} long range tensor part, as follows [14]:

$V_{res} = V_C + V_{S.l} + V_{ten}$ … (15)

3. Calculations

Shell model calculations for the FP-shell to be presented in this study for nucleus ^{58}Ni, in this case there are two neutrons outside the inert core ^{56}Ni which occupy the model space ($1F_{7/2}$, $2P_{3/2}$ and $2P_{1/2}$). Our resulting values as shown in fig (3) the fifth column were calculated using equation (5). The calculations are done as in the flowchart which is shown in fig (2).

The abnormal phenomena need an experience and "known how" that's related with the fp shell nuclei and their strange, mystery, abnormal, and unexpected results that make the calculations deviate from the experimental one and many efforts were exhausted in order to give a reasonable analysis and justifications about these cases. The single particle wave functions of the harmonic oscillator (HO) with size parameter b= 1.988 fm are used [15].

The energy levels in ^{58}Ni values were obtained by the shell model calculations that performed via the computer code OXBASH [16].

M3Y calculations are performed with a computer code written by Raad A. Radhi in FORTRAN language 90, modified by Firas Z. Majeed to produce realistic interaction [1].

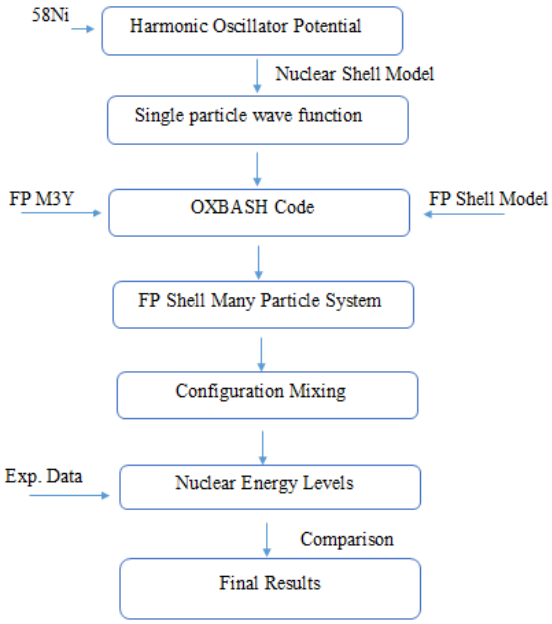

Fig. (2) Nuclear Energy Levels Calculations

4. Result and discussions

The result of energy levels spectra using (FP M3YE) interaction in comparison with experimental data, MSDI, RP and NZPSM is shown in Fig (3).

In the range between (0-2.5) MeV, the sequence of the levels are predicated, and the assigned J^π levels are coincided. Then in the range of energy between (2.5- 4) MeV some of assigned levels are predicated especially for the levels($2_2^+, 1_1^+, 0_2^+, 1_2^+, 4_2^+ \; and \; 2_5^+$), and some of assigned levels are displaced in some extents especially for the levels ($3_1^+, 2_3^+ \; and \; 2_4^+$) with deviation about ($\Delta = 0.3 \; MeV$).

Energy levels between (4- 5) MeV are showing a sequence of multi-polarity ($3_2^+ \; and \; 0_3^+$) are not arranged in order of sequence with experimental data. Levels spacing higher than MSDI, RP, and NZPSM gradually decreased.

Staying in Fig. (3), our model (FP M3YE) generate energy levels with high order ($J^\pi T$) more than that of MSDI, RP and NZPSM, and with an energy levels spacing higher than MSDI, RP, and NZPSM gradually decreased.

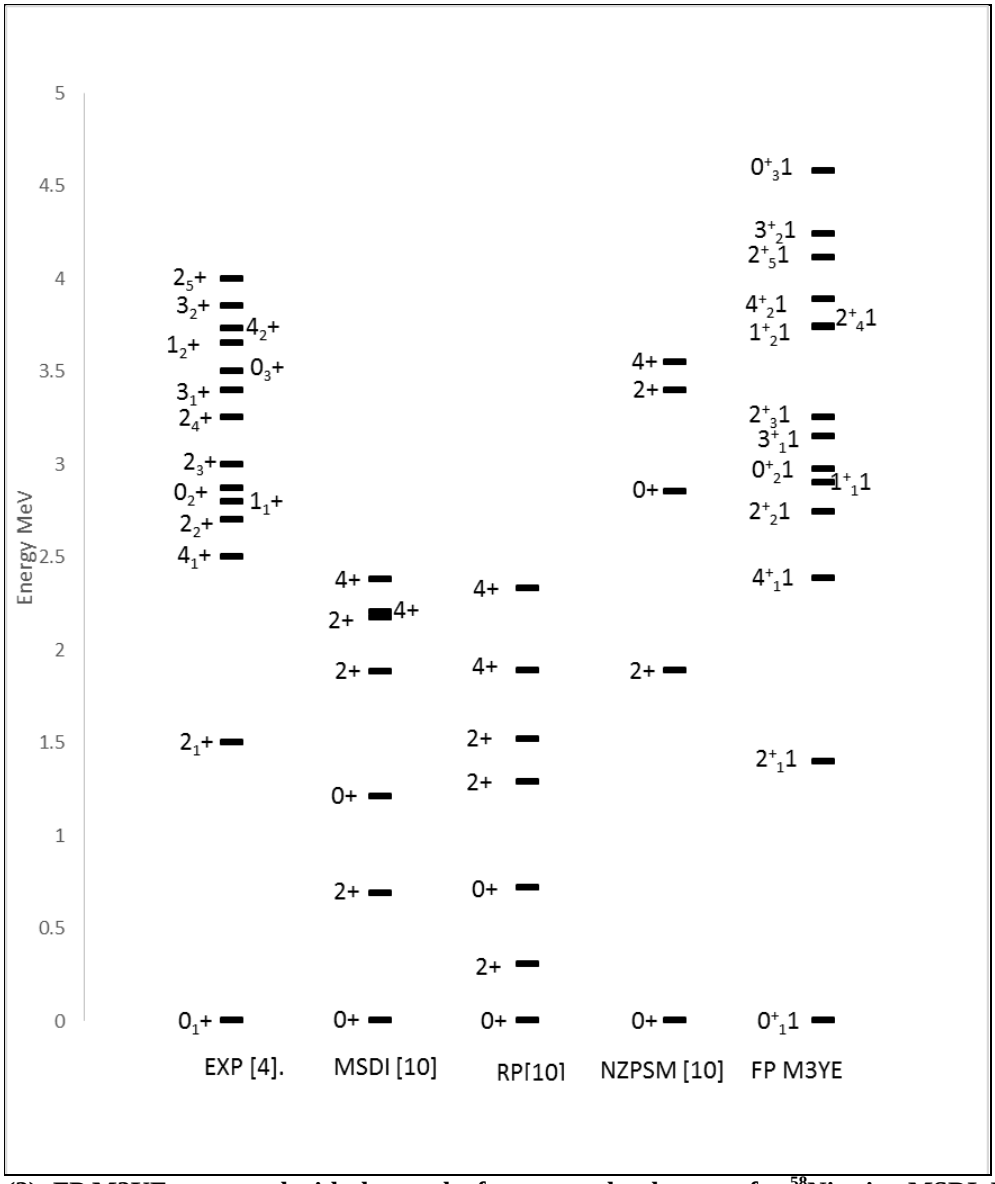

Fig (3): FP M3YE compared with the results for spectra level energy for ^{58}Ni using MSDI, RP, experimental and NZPSM method.

5. Conclusions

Realistic M3Y is very useful to reproduce energy levels in ^{58}Ni especially for the low laying states.

- In general energy levels with the use of modern realistic interaction are well consisted in some extent with exp. Data in comparison the former researchers.
- FP M3YE interaction will enhance the calculation where the particles in full FP-shell are included and make them occupying the low laying orbit ($1f_{7/2}$) and make the other active to reproduce nuclear properties.
- Even values of L (L= 0, 2, 4, 6…) are well consisted especially for low levels.
- Odd values of L (L= 1, 3, 5, 7…) needs more modifications in order to make these levels well consisted in comparison with experimental data.

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مستويات الطاقة النووية لنظير النيكل ^{58}Ni

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الملخص

تم حساب مستويات الطاقة والزخم الزاوي الكلي والتماثل لنيوكليونين متواجدة خارج القلب الخامل لنواة ^{56}Ni واللذان يحتلان القشرة النوويه FP بتطبيق انموذج القشرة النووي.

تم استخدام تفاعل FP M3YE لحساب مستويات الطاقة النوويه لنظير النيكل ^{58}Ni. وتم مقارنة النتائج مع القيم التجريبيه والنتائج النظرية لتفاعل مصفوفة دلتا المطور (MSDI) وجهد ريد (RP) و نموذج القشرة المزدوج الاصفري (NZPSM).

تم استخدام جهد المتذبذب التوافي لانشاء دالة الموجه للجسيم المنفرد في القشرة النووية FP باعتبار ان $^{56}_{28}Ni_{28}$ هو القلب الخامل.

و تم استخدام كود اوكسباش لغرض حسابات نموذج القشرة و ايضا تم مقارنة النتائج مع البيانات التجريبية.