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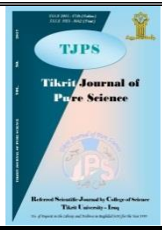

WES-2-Absorbing Submodules and WEQ-2-Absorbing Submodules

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ABSTRACT

In this research, we introduced and studied, the concepts of WES-2-absorbing submodules and WEQ-2-absorbing submodules as a stronger form of the concepts of weakly semi-2-absorbing submodules and weakly quasi-2-absorbing submodules respectively, and give their basic properties, examples and characterizations. On the otherhand, we studied the relationships among these concepts. Furthermore, we studied the behavior of these concepts in some classes of modules.

1. Introduction

Throughout this article, a ring R is commutative and Y is a unitary left R-module. A proper submodule A of an R-module Y is called a weakly semi-2-absorbing if, whenever $0 \neq \propto^2 y \in A$, with $\propto \in R, y \in Y$, implies that either $\propto y \in A$ or $\propto^2 \in [A:Y]$ [1], and a proper submodule A of an R-module Y is called a weakly quasi-2-absorbing, if whenever $0 \neq \propto \beta \gamma y \in A$,with $\alpha, \beta, \gamma \in R, y \in Y$, *imples that either* $\alpha\beta y \in A$ or $\propto \gamma y \in A$ or $\beta\gamma y \in A$ [2].It is clear that every weakly quasi-2-absorbing submodule is a weakly semi-2-absorbing submodule . Recall that an R-module Y is called multiplication if every submodule A of Y is of the form A= IY *for some* ideal I of R [3]. A proper submodule A of an R-module Y is called fully invariant if f(A)$\subseteq A$ for each f∈End(Y) [4]. A proper submodule B of an R-module Y is called stable if f(B)$\subseteq B$ for each R-homomorphism f:B→ Y, *and an* R-module Y is called fully stable if every submodule of Y is stable [4]. We proved that in the classes of multiplication modules , cyclic modules the concept of WES-2-absorbing and weakly semi-2-absorbing are equivalent . Also, in the class of scalar modules we prove that the concepts WEQ-2-absorbing submodule and weakly quasi-2-absorbing submodule are equivalents .

2. WES-2-Absorbing Submodules

This section devoted to intrduce, the concept of WES-2-absorbing submodules as a stronger form of Weakly semi 2-absorbing submodule, and give some choracterizations , properties and examples of this concept.

Definition (2.1)

A proper submodule A of an R-module Y is called a WES-2-absorbing submodule of Y , if whenever $0 \neq \Psi^2$(y)$\in A$, where $\Psi \in End(Y)$, y$\in Y$, implies that either $\Psi(y) \in A$ or $\Psi^2(Y) \subseteq A$. And an ideal I of a ring R is called a WES-2-absorbing if I is a WES-2-absorbing R-submodule of an R-module R.

Proposition (2.2)

Every WES-2-absorbing submodule of an R-module Y is a Weakly semi-2-absorbing submodule of Y.

Proof

Let A be a WES-2-absorbing submodule of Y, and $0 \neq \propto^2 y \in A$, $\propto \in R$, $y \in Y$. Assume that $\Psi: Y \to Y$ defined by $\Psi(y) = \propto y$ for each y$\in Y$, clearly $\Psi \in End(Y)$. Thus $0 \neq \propto^2 y = \Psi^2(y) \in A$, implies that either $\Psi(y) \in A$ or $\Psi^2(Y) \subseteq A$. Hence either $\propto y \in A$ *or* $\propto^2 \in [A:Y]$.

The converse of Proposition 2.2 is not true in general, as the following example shows:

Example (2.3)

Let Y=$Z\oplus Z$, $R = Z$ and $A = Z\oplus 10Z$. Clearly A is a weakly semi 2-absorbing submodule of Y. But A is not WES-2-absorbing submodule of Y, define Ψ:Y→ Y by $\Psi(x,y) = (y,x)$ for all x,y∈ Y . Clearly $\Psi \in End(Y)$, if (5,0)∈ Y , then $0 \neq \Psi^2(5,0) = (5,0) \in A$, but $\Psi(5,0) = (0,5) \notin A$ and $\Psi^2(Y) \nsubseteq A$.

The following results is a characterization of WES-2-absorbing submodules.

Theorem (2.4)

A proper sub module B of an R-module Y is a WES-2-absorbing if and only if $[\boldsymbol{B}:\boldsymbol{\Psi}^2(\boldsymbol{y})] = [\boldsymbol{B}:\boldsymbol{\Psi}(\boldsymbol{y})]$ for all nonzero $\boldsymbol{y} \in \boldsymbol{Y}$ and a nonzero $\boldsymbol{\Psi} \in \boldsymbol{End}(\boldsymbol{Y})$ or $\boldsymbol{\Psi}^2(\boldsymbol{Y}) \nsubseteq \boldsymbol{B}$.

Proof

(⇒) Suppose that $\Psi^2(Y) \nsubseteq B$. Clearly $[B:\Psi(y)] \subseteq [B:\Psi^2(y)]$, let $0 \neq s \in [B:\Psi^2(y)]$, implies that $0 \neq \Psi^2(sy) \in B$, but B is a WES-2-absorbing , and $\Psi^2(Y) \nsubseteq B$, then $\Psi(sy) = s\Psi(y) \in B$, it follows that s∈ $[B:\Psi(Y)]$.

(⇐) Let $0 \neq \Psi^2(y) \in B$, where Ψ is a non zero element in End(Y) ,and y in Y . Then either $[B:\Psi^2(y)] = [B:\Psi(y)]$ *or* $\Psi^2(Y) \subseteq B$. If $[B:\Psi^2(y)] = [B:\Psi(y)]$, then $[B:\Psi^2(y)] = R$. So $[B:\Psi(y)] = R$, and hence $\Psi(y) \in B$.

Proposition (2.5)

Let B be a proper submodule of multiplication R-module Y . Then B is a WES-2-absorbing if and only if B is a Weakly semi-2-absorbing .

Proof

(⇒) By Proposition 2.2 .

(⇐) Let $0 \neq \Psi^2(y) \in B$, where 0≠ $\Psi \in End(Y)$, y∈ Y . Since Y is a multiplication , then by [3, lemma 1.4] ,there exists s∈ R , such that $\Psi(y) = sy$ for all y∈ Y . That is $0 \neq \Psi^2(y) = s^2y \in B$, implies that either sy∈ B or $s^2 \in [B:Y]$ .Thus , either sy∈ B *or* $s^2(Y) \subseteq Y$, hence either $\Psi(y) \in B$ or $\Psi^2(Y) \subseteq B$.

Since cyclic R-module is multiplication module, then we have the following corollary.

Corollary (2.6)

Let B be a proper submodule of cyclic R-module Y. Then B is a WES-2-absorbing submodule of Y if and only if B is a weakly semi 2-absorbing submodule of Y.

Proposition (2.7)

Let B be proper submodule of a fully stable module Y, and $\Psi: Y \to Y$ be a nonzero R-homomorphism. If B is a WES-2-absorbing submodule of Y, then $\Psi^{-1}(B)$ is a WES-2-absorbing submodule of Y.

Proof

Clearly $\Psi^{-1}(B)$ is a proper submodule of Y . Let $0 \neq \Phi^2(y)\epsilon\Psi^{-1}(B)$, where Φ be a nonzero element in End(Y) . Then $0 \neq \Psi(\Phi^2(y)) \in B$. But Y is fully stable, then by [4,Prop. 2.1], End(Y) is commutative. Thus $0 \neq \Psi(\Phi^2(y)) = \Phi^2(\Psi(y))\epsilon B$, hence either $\Phi(\Psi(y)) \in B$ *or* $\Phi^2(Y) \subseteq B$. If $\Phi(\Psi(y)) \in B$, then $\Psi(\Phi(y)) \in B$, so $\Phi(y) \in \Psi^{-1}(B)$. If $\Phi^2(Y) \subseteq B$, and Y is a fully stable , then B is a stable submodule of Y , then by [4, p.7] B is a fully invariant , that is $\Psi(B) \subseteq B$, implies that B⊆ $\Psi^{-1}(B)$, hence $\Phi^2(Y) \subseteq B \subseteq \Psi^{-1}(B)$. So $\Psi^{-1}(B)$ is a WES-2-absorbing submodule of Y .

Proposition (2.8)

Let A,B be a proper submodule of an R-module Y , with A⊆ B , and A is a fully invariant submodule of Y . If B/A is a WES-2-absorbing submodule of Y/A, then B is a WES-2-absorbing submodule of Y .

Proof

Suppose that $0 \neq \Psi^2(y) \in B$, $0 \neq \Psi \in End(Y)$, y∈ Y , and let $\Phi: Y/A \to Y/A$ be a nonzero R-homomorphism , defined by $\Phi(y + A) = \Psi(y) + A$. Since A is a fully invariant, then clearly Φ is a well defined homomorphisim. Now, $0 \neq \Phi^2(y + A) = \Phi^2(y) + A = \Phi(\Psi(y) + A) = \Psi^2(y) + A \in B/A$. It follows that either $\Phi(y + A) \in \frac{B}{A}$ or $\Phi^2(Y/A) \subseteq B/A$.That is $\Phi(y + A) = \Psi(y) + A \in B/A$ or $\Psi^2(Y) \subseteq B$ .Therefore either $\Psi(y) \in B$ or $\Psi^2(Y) \subseteq B$.

Proposition (2.9)

Let B a WES-2-absorbing submodule of an R-module Y , then [B:Y] is a WES-2-absorbing ideal of R .

Proof

Since B is a WES-2-absorbing submodule of Y, then by Proposition 2.2 B is s weakly semi-2-absorbing submodule in Y. Hence by [1, prop. 2.4], [B:Y] is weakly semi-2-absorbing ideal of R . Since R is cyclic R-module , then by Corollary 2.6, [B:Y] is a WES-2-absorbing submodule of Y .

Proposition (2.10)

Let Y be a cyclic R-module , and B be a proper submodule of Y . Then B is a WES-2-absorbing submodule of Y if and only if [B:Y] is a WES-2-absorbing ideal of R.

Proof

(⇒) Follows by Proposition 2.9 .

(⇐) Follows by [1,Coro. 2.5] , and Proposition 2.2 .

We end this section by the following proposition .

Proposition (2.11)

Let Y=$Y_1\oplus Y_2$ be a multiplication R-module, where Y_1, Y_2 are cyclic R-module, and A,B are proper submodule of Y_1 and Y_2 respectively . Then

1- A is a WES-2-absorbing submodule of Y_1 if and only if A$\oplus Y_2$ is a WES-2-absorbing submodule of Y.

2- B is a WES-2-absorbing submodule of Y_2 if and only if $Y_1 \oplus B$ is a WES-2-absorbing submodule of Y.

Proof

(1) **(⇒)** Since A is a WES-2-absorbing submodule of Y_1 **,** then by Proposition 2.2 A is a weakly semi-2-absorbing submodule of Y_1 **.** Thus by [1,Prop**.** 2.16], A$\oplus Y_2$ is a weakly semi-2-absorbing submodule of Y . But Y is multiplication, then by Proposition2.5, A$\oplus$ Y_2 a WES-2-absorbing submodule of Y .

(⇐) Since A$\oplus$ Y_2 is a WES-2-absorbing submodule of Y , then by Proposition 2.2, A$\oplus Y_2$ is a Weakly semi-2-absorbing submodule of Y. Hence by [1, Prop. 2.16] we have A is a weakly semi 2-absorbing submodule of Y_1 . Since Y_1 is cyclic , then by

Corollary 2.6, A is WES-2-absorbing submodule of Y_1.

In the same way , we can prove (2) .

3- WEQ-2-Absorbing Submodules

This section deals with the concept of a WEQ-2-absorbing submodule , which is a stronger form of Weakly quasi-2-absorbing submodule. We give some basic properties and characterizations of this concept.

Definition (3.1)

A proper submodule B of an R-module Y is called a WEQ-2-absorbing submodule , if whenever $0 \neq \Psi o \Phi o \Gamma(y) \in B$ for all nonzero, $\Psi, \Phi, \Gamma \in End(Y)$, and $y \in Y$, implies that either $(\Psi o \Phi)(y) \in B$ or $(\Psi o \Gamma)(y) \in B$ or $(\Phi o \Gamma)(y) \in B$. And a proper ideal I of a ring R is called WEQ-2-absorbing ideal of R, if I is a WEQ-2-absorbing R-submodule of an R-module R.

The following proposition, gives the relation of a WEQ-2-absorbing submodules , with a weakly quasi-2-absorbing submodules.

Proposition (3.2)

Let Y be an R-module , and A is a WEQ-2-absorbing submodule of Y. Then A is a weakly quasi-2-absorbing submodule of Y .

Proof

Let $0 \neq \alpha\beta\gamma y \in A$, with $\alpha, \beta, \gamma \in R$, $y \in Y$, and let $\Psi, \Phi, \Gamma: Y \longrightarrow Y$ defined by $\Psi(y) = \alpha y$, $\Phi(y) = \beta y$, and $\Gamma(y) = \gamma y$ for all $y \in Y$. Clearly $\Psi, \Phi, \Gamma \in End(Y)$. Now, $0 \neq \alpha\beta\gamma y = (\Psi o \Phi o \Gamma)(y) \in A$. Since A is a WEQ-2-absorbing , then either $(\Psi o \Phi)(y) \in A$ or $(\Psi o \Gamma)(y) \in A$ or $(\Phi o \Gamma)(y) \in A$. Hence either $\alpha\beta y \in A$ or $\alpha\gamma y \in A$ or $\beta\gamma y \in A$. Thus A is a weakly quasi-2-absorbing submodule of Y .

The converse of Proposition 3.2 is not true in general , as the following example shows:

Example (3.3)

Let $Y = Z \oplus Z$, R=Z , $A = 5Z \oplus (0)$. Clearly A is a weakly quasi 2-absorbing submodule of Y . But A is not WEQ-2-absorbing submodule of Y . Since if $\Psi, \Phi, \Gamma: Y \longrightarrow Y$ defined by $\Psi(x,y) = (y,x)$, $\Phi(x,y) = (0,x)$ and $\Gamma(x,y) = (y,0)$ for all $x, y \in Z$. Clearly $\Psi, \Phi, \Gamma \in End(Y)$. Now, $0 \neq (\Psi o \Phi o \Gamma)(1,5) = (5,0) \in A$ but $(\Psi o \Phi)(1,5) = \Psi(0,1) = (1,0) \notin A$, and $(\Psi o \Gamma)(1,5) = \Psi(5,0) = (0,5) \notin A$ and $(\Phi o \Gamma)(1,5) = \Phi(5,0) = (0,5) \notin A$. Thus A is not WEQ-2-absorbing submodule of Y .

Recall that An R-module Y is called a scalar module, if every $f \in End(Y)$, there exists $r \in R$ such that f(y)= ry for each $y \in Y$ [5].

The following proposition shows that in the class of a scalar modules WEQ-2-absorbing submodule and weakly quasi 2-absorbing submodule are equivalent.

Proposition (3.4)

Let B be a proper submodule of a scalar R-module Y. Then B is a WEQ-2-absorbing submodule of Y if and only if B is a weakly quasi 2-absorbing submodule of Y.

Proof

($\Longrightarrow$) Follows by Proposition 3.2.

($\Longleftarrow$) Let $0 \neq (\Psi o \Phi o \Gamma)(y) \in B$, where Ψ, Φ, Γ are nonzero elements in End(Y) , $y \in Y$. Since Y is a scalar module, then there exists $r, s, t \in R$ such that $\Psi(y) = ry$, $\Phi(y) = sy$, $\Gamma(y) = ty$ for all $y \in Y$.Thus $0 \neq (\Psi o \Phi o \Gamma)(y) = rsty \in B$. But B is a weakly quasi-2-absorbing in Y , then either $rsy \in B$ or $rty \in B$ or $sty \in B$. It follows that either $(\Psi o \Phi)(y) \in B$ or $(\Psi o \Gamma)(y) \in B$ or $(\Phi o \Gamma)(y) \in B$.

The following propositions are characterizations of a WEQ-2-absorbing submodules.

Proposition (3.5)

Let Y be an R-module , B a proper submodule of Y . Then B is a WEQ-2-absorbing submodule of Y if and only if for each nonzero $\Psi, \Phi, \Gamma \in End(Y)$, with $\Psi o \Phi o \Gamma(y) \neq 0$, $[B:_Y \Psi o \Phi o \Gamma] = [B:_Y \Psi o \Phi] \cup [B:_Y \Psi o \Gamma] \cup [B:_Y \Phi o \Gamma]$.

Proof

($\Longrightarrow$) Clearly $[B:_Y \Psi o \Phi] \cup [B:_Y \Psi o \Gamma] \cup [B:_Y \Phi o \Gamma] \subseteq [B:_Y \Psi o \Phi o \Gamma]$.

Now, let $y \in [B:_Y \Psi o \Phi o \Gamma]$, then $0 \neq (\Psi o \Phi o \Gamma)(y) \in B$, implies that either $(\Psi o \Phi)(y) \in B$ or $(\Psi o \Gamma)(y) \in B$ or $(\Phi o \Gamma)(y) \in B$, it follows that either $y \in [B:_Y \Psi o \Phi]$ or $y \in [B:_Y \Psi o \Gamma]$ or $y \in [B:_Y \Phi o \Gamma]$. Hence $y \in [B:_Y \Psi o \Phi] \cup [B:_Y \Psi o \Gamma] \cup [B:_Y \Phi o \Gamma]$.

($\Longleftarrow$) Suppose that $[B:_Y \Psi o \Phi o \Gamma] = [B:_Y \Psi o \Phi] \cup [B:_Y \Psi o \Gamma] \cup [B:_Y \Phi o \Gamma]$, with $\Psi o \Phi o \Gamma \neq 0$. Let $0 \neq \Psi o \Phi o \Gamma(y) \in B$ for nonzero $\Psi, \Phi, \Gamma \in End(Y)$, $y \in Y$, implies that $y \in [B:_Y \Psi o \Phi o \Gamma] = [B:_Y \Psi o \Phi] \cup [B:_Y \Psi o \Gamma] \cup [B:_Y \Phi o \Gamma]$. It follows that $y \in [B:_Y \Psi o \Phi]$ or $y \in [B:_Y \Psi o \Gamma]$ or $y \in [B:_Y \Phi o \Gamma]$. Hence either $(\Psi o \Phi)(y) \in B$ or $(\Psi o \Gamma)(y) \in B$ or $(\Phi o \Gamma)(y) \in B$. Thus B is a WEQ-2-absorbing submodule of Y .

Proposition (3.6)

Let B be a proper submodule of an R-module Y. Then B is a WEQ-2-absorbing submodule of Y if and only if for each a nonzero $\Psi, \Phi \in End(Y)$, and $y \in Y$ with $0 \neq (\Psi o \Phi)(y) \notin B$, we have $[B:(\Psi o \Phi)(y)] = [B:\Psi(y)] \cup [B:\Phi(y)]$.

Proof

($\Longrightarrow$) Let $0 \neq h \in [B:(\Psi o \Phi)(y)]$, implies that $0 \neq (h o \Psi o \Phi)(y) \in B$. But B is a WEQ-2-absorbing submodule of Y , and $(\Psi o \Phi)(y) \notin B$, then either $(h o \Psi)(y) \in B$ or $(h o \Phi)(y) \in B$. That is either $h \in [B:\Psi(y)]$ or $h \in [B:\Phi(y)]$, then $h \in [B:\Psi(y)] \cup [B:\Phi(y)]$, it follows that $[B:\Psi o \Phi(y)] \subseteq [B:\Psi(y)] \cup [B:\Phi(Y)]$. Clearly $[B:\Psi(y)] \cup [B:\Phi(y)] \subseteq [B:\Psi o \Phi(y)]$.

($\Longleftarrow$) suppose that Ψ, Φ are nonzero elements in End(Y) , $y \in Y$ with $0 \neq \Psi o \Phi(y) \notin B$, we have $[B:(\Psi o \Phi)(y)] = [B:\Psi(y)] \cup [B:\Phi(y)]$,and let $0 \neq (\Gamma o \Psi o \Phi)(y) \in B$ where Φ, Ψ, Γ are nonzero elements in End(Y) , $y \in Y$ and $(\Psi o \Phi)(y) \notin B$. It follows that $\Gamma \in [B:\Psi o \Phi(y)]$, implies that $\Gamma \in [B:\Psi(y)]$ or $\Gamma \in [B:\Phi(Y)]$, implies that either $\Gamma o \Psi(y) \in B$ or $\Gamma o \Phi(y) \in B$. Thus B is a WEQ-2-absorbing submodule in Y .

Proposition (3.7)

Let $\{Ai\}_{i \in \Lambda}$ be a family of a chain WEQ-2-absorbing submodules of an R-module Y. Then $\cap_{i \in \Lambda} Ai$ is a

WEQ-2-absorbing submodule of Y.

Proof

Let $0 \neq (\Psi o \Phi o \Gamma)(y) \in \bigcap_{i\in\Lambda} Ai$, where Ψ, Φ, Γ are nonzero elements in End(Y) , y$\in Y$, and $\Psi o\Phi(y) \notin \bigcap_{i\in\Lambda} Ai$, and $(\Psi o \Gamma)(y) \notin \bigcap_{i\in\Lambda} Ai$. Then there exists $t, e \in \Lambda$ such that $(\Psi o \Phi)(y) \notin A_t$, $(\Psi o \Gamma)(y) \notin A_e$. Hence for $A_s \subseteq A_t$ and for $A_d \subseteq A_e$, we have $(\Psi o \Phi)(y) \notin A_s$ and $(\Psi o \Gamma)(y) \in A_d$. Thus for every A_r such that $A_r \subseteq A_t$ and $A_r \subseteq A_e$, we get $(\Phi o \Gamma)(y) \subseteq A_r$. Hence $(\Phi o \Gamma)(y) \in \bigcap_{i\in\Lambda} Ai$. $\bigcap_{i\in\Lambda} Ai$ is a WEQ-2-absorbing submodule of Y .

Proposition (3.8)

Let Y be an R-module , and B a proper submodule of Y . Then B is a WEQ-2-absorbing submodule of Y. if and only if $0 \neq (\Psi o \Phi o \Gamma)(A) \subseteq B$ for nonzero $\Psi, \Phi, \Gamma \in End(Y)$, and a nonzero submodule A of Y , implies that either $(\Psi o \Phi)(A) \subseteq B$ or $(\Psi o \Gamma)(A) \subseteq B$ or $(\Phi o \Gamma)(A) \subseteq B$.

Proof

It follows directly from definition of WEQ-2-absorbing submodule.

Proposition (3.9)

Let $f: Y \longrightarrow Y$ be an R-epimorphism , and A a fully invariant submodule of Y. If A is a WEQ-2-absorbing submodule of Y , then $f^{-1}(A)$ is a WEQ-2-absorbing submodule of Y .

Proof

Let $0 \neq (\Psi o \Phi o \Gamma)(y) \in f^{-1}(A)$, where Ψ, Φ, Γ are nonzero elements in End(Y) , y$\in Y$, implies that $0 \neq f o \Psi o \Phi(\Gamma(y)) \in A$. But A is a WEQ-2-absorbing submodule of Y, then either $f o \Psi(\Gamma(y)) \in A$ or $f o \Phi(\Gamma(y)) \in A$ or $\Psi o \Phi(\Gamma(y)) \in A$. It follows that :

If $(f o \Psi)(\Gamma(y)) \in A$, implies that $(\Psi o \Gamma)(y) \in f^{-1}(A)$.

If $(f o \Phi)(\Gamma(y)) \in A$, implies that $(\Phi o \Gamma)(y) \in f^{-1}(A)$.

If $0 \neq (\Psi o \Phi)(\Gamma(y)) \in A$, and A is a WEQ-2-absorbing submodule of Y , then either $(\Psi o \Gamma)(y) \in$ A or $(\Phi o \Gamma)(y) \in$ A or $(\Psi o \Phi)(y) \in$ A. Since A is a fully invariant , so if $(\Psi o \Gamma)(y) \in A$, implies that $f o \Psi o \Gamma(y) \in f(A) \subseteq A$, implies that $(\Psi o \Gamma)(y) \in f^{-1}(A)$. If $(\Phi o \Gamma)(y) \in A$, implies that $(f o \Phi o \Gamma)(y) \in f(A) \subseteq A$, then $(\Phi o \Gamma)(y) \in f^{-1}(A)$. If $(\Psi o \Phi)(y) \in A$, implies that $(f o \Psi o \Phi)(y) \in f(A) \subseteq A$, hence $(\Psi o \Phi)(y) \in f^{-1}(A)$. Thus $f^{-1}(A)$ is a WEQ-2-absorbing submodule of Y.

Proposition (3.10)

Let Y be an R-module , and A is a proper submodule of Y and Ψ:Y$\longrightarrow Y'$ be an R-epimorphism with $ker\Psi \subseteq A$. If A is a WEQ-2-absorbing submodule of Y , then $\Psi(A)$ is a WEQ-2-absorbing submodule Y' , where Y' is a Y-projective module .

Proof

Suppose that $0 \neq (f o g o h)(y') \in \Psi(A)$, where f, g, h are nonzero elements in End(Y'), $y' \in Y'$. Since Ψ is an epimorphism , then there exists $y \in Y$ such that $\Psi(y) = y'$, and since Y' is a Y-projective , then there exists $f_1, f_2, f_3: Y' \longrightarrow Y$ such that $\Psi o f_1 = f$, $\Psi o f_2 = g$, $\Psi o f_3 = h$. Now , we have $f_1 o \Psi \in End(Y)$ and $f_2 o \Psi \in End(Y)$ and $f_3 o \Psi \in End(Y)$. Also, we have $0 \neq (f o g o h)(y') = (\Psi o f_1) o (\Psi o f_2) o (\Psi o f_3)(y') \in \Psi(A)$, implies that $0 \neq (f o g o h)(y_1) = \Psi[f_1 o \Psi o f_2 o \Psi o f_3(y')] \in \Psi(A)$, then there exists a nonzero element x$\in A$ such that $\Psi(f_1 o \Psi o f_2 o \Psi o f_3(y') = \Psi(x)$, implies that $0 \neq f_1 o \Psi o f_2 o \Psi o f_3(y') \in ker\Psi \subseteq A.$ Hence $0 \neq f_1 o \Psi o f_2 o \Psi f_3 o \Psi(y) \in A.$ That is $0 \neq (f_1 o \Psi) o (f_2 o \Psi) o (f_3 o \Psi)(y) \in A$. But A is a WEQ-2-absorbing submodule of Y, then either $(f_1 o \Psi) o (f_2 o \Psi)(y) \in A$ or $(f_1 o \Psi) o (f_3 o \Psi)(y) \in A$ or $(f_2 o \Psi) o (f_3 o \Psi)(y) \in A$. It follows that either $\Psi o f_1 o \Psi o f_2 o \Psi(y) \in \Psi(A)$, implies that $(f o g)(y') \in \Psi(A)$ or $\Psi o f_1 o \Psi o f_3 o \Psi(y) \in \Psi(A)$, implies that $(f o h)(y') \in \Psi(A)$ or $\Psi o f_2 o \Psi o f_3 o \Psi(y) \in \Psi(A)$, implies that $(g o h)(y') \in \Psi(A)$. Hence $\Psi(A)$ is a WEQ-2-absorbing submodule of Y .

Proposition (3.11)

Let A be a WEQ-2-absorbing submodule of an R-module Y , and B is a Y-injective submodule of Y . Then either B$\subseteq A$ or B$\cap A$ is a WEQ-2-absorbing submodule of B.

Proof

Assume that B$\nsubseteq A$, then B$\cap A$ is a proper submodule of B. Let $0 \neq (\Psi o \Phi o \Gamma)(y) \in B \cap A$, where, $y \in B$ and Ψ, Φ, Γ are nonzero elements in End(B) . Since B is a Y-injective , then there exists $f_1, f_2, f_3: Y \longrightarrow B$ such that $f_1 o i = \Psi$, $f_2 o i = \Phi$, and $f_3 o i = \Gamma$, where i is the inclusion map from B into Y. Clearly $f_1, f_2, f_3 \in End(Y)$. Now, $0 \neq (\Psi o \Phi o \Gamma)(y) = [(f_1 o i) o (f_2 o i) o (f_3 o i)](y) = (f_1 o i o f_2 o i o f_3)(y) = f_1 o f_2 o f_3(y) \in A$. But A is a WEQ-2-absorbing submodule of Y, then either $f_1 o f_2(y) \in A$ or $f_1 o f_3(y) \in A$ or $f_2 o f_3(y) \in A$. It follows that either $(\Psi o \Phi)(y) \in A$ or $(\Psi o \Gamma)(y) \in A$ or $(\Phi o \Gamma)(y) \in A.$ Since $\Psi, \Phi, \Gamma \in End(B)$, then we have either $(\Psi o \Phi)(y) \in B \cap A$ or $(\Psi o \Gamma)(y) \in B \cap A$ or $(\Phi o \Gamma)(y) \in B \cap A$. Hence B$\cap A$ is a WEQ-2-absorbing submodule of B.

Proposition (3.12)

Let Y be an R-module, and A is a WEQ-2-absorbing submodule of Y, then A is a WES-2-absorbing submodule of Y.

Proof

Let $0 \neq \Psi^2(y) \in A$, with Ψ be a nonzero element in End(Y),y$\in Y$.That is $0 \neq (\Psi o \Psi)$oI$(y) \in A$, where I is the identity element in End(Y) . Since A is a WEQ-2-absorbing submodule, then either $\Psi o \Psi(y) \in A$ or ΨoI$(y) \in A$ for each y$\in Y$. Hence either $\Psi(y) \in A$ or $\Psi^2(Y) \subseteq A$, that is A is a WES-2-absorbing submodule of Y.

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المقاسات الجزئية المستحوذة على−2 من النمط WES والمقاسات الجزئية المستحوذة على−2 من النمط WEQ

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الملخص

في هذا البحث قدمنا مفاهيم المقاسات الجزئية المستحوذة على −2 من النمط WES والمقاسات الجزئية المستحوذة على −2 من النمط WEQ كمفاهيم اقوى من المقاسات الجزئية المستحوذة على − 2 الشبه ضعيفة والمقاسات الجزئية المستحوذة على − 2 الظاهرية الضعيفة على التوالي واعطينا بعض الصفات الأساسية والامثلة والمكافئات لهما. من ناحية أخرى درسنا شكل العلاقة بين هذه المفاهيم مع بعضها. بالإضافة الى ذلك درسنا سلوك هذه المفاهيم في بعض صفوف المقاسات الأخرى.