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ON Supra $\alpha-$ Compactness In Supra topological Spaces

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ABSTRACT

The purpose of this paper is to introduce the concept of strongly supra $\alpha-$ continuous function, perfectly supra $\alpha-$ continuous function and totally supra $\alpha-$ continuous function, The relationships among these functions are studied., and investigated some properties of them. Also we introduced the concepts of supra $\alpha-$ compact space, supra $\alpha-$ Lindelof spaces and countably supra $\alpha-$ compact spaces. Some basic properties are proved. At last the relationships among supra $\alpha-$ open, supra $\alpha-$ continuous maps and supra $\alpha-$ irresolute maps in supra topological spaces .

1. Introduction

In 1983, Mashhour, A.S. et al [1] introduced the supra topological spaces and studied $S-$ continuous maps and S^*- continuous maps. In 2008, Devi, R. et al [2] introduced and studied a class of sets and maps between topological spaces called supra $\alpha-$ open sets and $S\alpha-$ continuous maps. In 2008, Jassim, T.H. [3] came out with the concept of supra compactness in supra topological spaces. respectively. In 2012, Sekar, S. et al [4] introduced and investigated a new class of sets and function between topological spaces called supra $I-$ open sets and supra $I-$ continuous functions respectively. In 2013, Mustafa, J.M. [5] came out with the concept of supra $b-$ compact and supra $b-$ Lindelof spaces. In 2017, Krishnaveni and Vigneshwaran [6] came out with supra $b\mathcal{T}-$ closed sets and gave their properties. In 2018, Latif, R.M. [7] came out with the concept of supra $I-$ compactness and supra $I-$ connectdness. Now the study brings up with the new concepts of supra $\alpha-$ compact, countably supra $\alpha-$ compact, supra $\alpha-$ Lindelof spaces and present several properties and characteristics of these concepts.

2. Preliminaries

We recall some definitions which are needed in this work .

Definition 2.1. [1] Let X be a non-empty set and Let $\mu \subseteq P(X) = \{A : A \subseteq X\}$. Then μ is called a supra topology on X if $\varphi \in \mu, X \in \mu$ and for all $\gamma \subseteq \mu$, it implies that $\cup_{\gamma} \in \mu$. The pair(X,μ) is called a supra topological space. Each element $A \in \mu$ is called a supra open set in (X,μ) and the complement of A denoted by A^c =X – A is called a supra closed set in (X,μ) .

Definition 2.2. [1] Let (X,μ) be a supra topological space. The supra closure of a set A is denoted by supra $-cl(A) = \{B \subseteq X : B$ is a supra closed set in X such that $A \subseteq B\}$.

The supra interior of a set A is denoted by supra – $Int(A)$ and is defined by supra $-Int(A) = \{U \subseteq X : U$ is a supra open set in X such that $U \subseteq A\}$.

Definition 2.3. [1] Let $(X,\mathcal{T})$ be a topological space and μ be a supra topology on X . We call μ a supra topology associated with $\mathcal{T}$ if $\mathcal{T} \subseteq \mu$.

Definition 2.4. [2] Let (X,μ) be a supra topological space. A subset A of X is called a supra $\alpha-$ open set in X if $A \subseteq$ supra – Int [supra – cl [supra – Int (A)]] . The complement of supra $\alpha-$ open set is called a supra $\alpha-$ closed set .

Definition 2.5. [2] Let (X,μ) be a supra topological space . The supra $\alpha-$ closure of a set A is denoted by supra $-\ \alpha-cl\ (A)$, and is defined as given in the following :

Supra $-\alpha - cl\ (A) = \{B \subseteq X : B$ is supra $\alpha-$ closed set in X such that $A \subseteq B\}$.

The supra $\alpha-$ interior of a set A is denoted by supra $-\ \alpha - Int\ (A)$, and is defined by supra

supra $\alpha-$ open subset of Y is supra $\alpha-$ open subset of X .

Definition 3.2 A function $f:(X,\mu) \to (Y,\mu^*)$ is called strongly supra $\alpha-$ continuous if the inverse image of every supra $\alpha-$ open subset of Y is supra open in X .

Definition 3.3 A function $f:(X,\mu) \to (Y,\mu^*)$ is called perfectly supra $\alpha-$ continuous if the inverse image of every supra $\alpha-$ open subset of Y is both supra open and supra closed in X .

Definition 3.4 A function $f:(X,\mu) \to (Y,\mu^*)$ is called totally supra $\alpha-$ continuous if the inverse image of every supra open set V in Y is both supra $\alpha-$ closed and supra $\alpha-$ open in X .

Theorem 3.5.[2] Every continuous function is supra $\alpha-$ continuous functions .

Proof : Let $(X,\mathcal{T}) \to (Y,\mathcal{T}^*)$ be two topological spaces and μ and μ^* be associated supra topologies with $\mathcal{T}$ and $\mathcal{T}^*$ respectively. Let $f:X \to Y$ be a continuous function . Therefore $f^{-1}(A)$ is an open set in X for each open set A in Y . But, μ is associated with $\mathcal{T}$. That is $\mathcal{T} \subseteq \mu$. This implies that $f^{-1}(A)$ is a supra open set in X. Since every supra open set is supra $\alpha-$ open set, this implies $f^{-1}(V)$ is supra $\alpha-$ open in X. Hence f is supra $\alpha-$ continuous function .

The converse of the above theorem is not true as shown in the following example .

Example 3.6.[2] Let $X = \{a,b,c\}$ and $\mathcal{T} = \{\varphi, X, \{a,b\}\}$ be a topology on X . The supra topology μ is defined as follows, $\mu = \{\varphi, X, \{a\}, \{a,b\}\}$. Suppose that $f:X \to X$ is a function defined as follows : $f(\{a\}) = b$, $f(\{b\}) = c$, $f(\{c\}) = a$. The inverse image of the open set $\{a,b\}$ is $\{a,c\}$ which is not an open set but it is supra $\alpha-$ open . Then f is supra $\alpha-$ continuous but not continuou .

Theorem 3.7 Every perfectly supra α – continuous function is strongly supra α – continuous function .

Proof : Let $f:(X,\mu) \to (Y,\mu^*)$ be a perfectly supra α – continuous function, Let V be supra α – open set in (Y,μ^*). Since f is perfectly supra α – continuous function $f^{-1}(V)$ is both supra open and supra closed in (X,μ) . Therefore f is strongly supra α – continuous function .

The converse of the above theorem need not be true. It is shown by the following example .

Example 3.8 Let $X = Y = \{a,b,c\}$, $\mu = \{\varphi, X, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$, $\mu^* = \{\varphi, Y, \{c\}, \{a,b\}\}$. $f:(X,\mu) \to (Y,\mu^*)$ be the function defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Here f is strongly supra α – continuous but not perfectly supra continuous, since $\{c\}$ is supra α – open in Y but $f^{-1}\{c\} = \{c\}$ is supra open set but not supra closed in X .

Theorem 3.9 Let $f:(X,\mu) \to (Y,\mu^*)$ be strongly supra α – continuous and $g:(Y,\mu^*) \to (Z,\mu^{**})$ be strongly supra α – continuous then their composition

– α – $Int\,(A) = \{U \subseteq X : U$ is supra $\alpha-$ open set in X such that $U \subseteq A\}$. clearly supra – α – $cl\,(A)$ is a supra $\alpha-$ closed set. supra – α – $Int\,(A)$ is supra $\alpha-$ open set .

Throughout this paper, $(X,\mathcal{T})$ and $(Y,\mathcal{T}^*)$ will denote topological spaces and we will denote by μ and μ^* to be their associated supra topologies with $\mathcal{T}$ and $\mathcal{T}^*$ respectively such that $\mathcal{T} \subseteq \mu$ and $\mathcal{T}^* \subseteq \mu^*$.

Theorem 2.6. [2] Let (X,μ) be a supra topological space . Then every supra open set in X is supra $\alpha-$ open set in X .

The converse of the theorem (2.6) need not be true as shown by the following example .

Example 2.7. [2] Suppose $X = \{a,b,c\}$ and have the supra topology $\mu = \{\varphi, X, \{a\}\}$. The set $\{a,b\} \notin \mu$, so the set $\{a,b\}$ is not supra open set in (X,μ). Now since it clealy follows that supra – Int [supra – cl [supra – Int $(\{a,b\})$]] = supra – Int [supra – cl $(\{a\})$] = supra – Int [(X)] $= X$. Therefore it follows that $\{a,b\}$ is a supra α–open set in (X,μ).

Theorem 2.8. [2] (i). Arbitrary union of supra $\alpha-$ open sets is always a supra $\alpha-$ open set .

(ii). Finite intersection of supra $\alpha-$ open sets may fail to be a supra $\alpha-$ open set .

Proof : (i). Let (X,μ) be a supra topological space. Let $\psi = \{S_i : i \in I\}$ be afamily of supra $\alpha-$ open sets in X. Let $S = \cup\, \psi = \cup\, \{S_i : i \in I\}$. Since for each $i \in I$, S_i is supra $\alpha-$ open set. Hence it follows that $S_i \subseteq$ supra – Int [supra – cl [supra – Int (S_i)]] $\subseteq$ supra – Int [supra – cl [supra – Int (S)]], for all $i \in I$. So $S_i \subseteq$ supra – Int [supra – cl [supra – Int (S)]], for all $i \in I$. Therefore clearly it follows that $S = \cup_{i \in I} S_i \subseteq$ supra – Int [supra – cl [supra – Int (A)]]. Thus we conclude that S is supra $\alpha-$ open set .

(ii). Let $X = \{a,b,c\}$ and $\mu = \{\varphi, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$ be a supra topology on X. Then $\{a,b\}$ and $\{b,c\}$ are supra $\alpha-$ open sets but their intersection $\{b\}$ is not a supra $\alpha-$ open set .

Theorem 2.9. [2] (i). The arbitrary intersection of supra $\alpha-$ closed sets is always supra $\alpha-$ closed .

(ii). A finite union of supra $\alpha-$ closed sets may fail to be supra $\alpha-$ closed set .

Proof : (i) follows from theorem 2.8 (i) .

(ii). Let $X = \{1,2,3,4,5\}$ and $\mu = \{\varphi, X, \{1,2\}, \{1,2,3\}, \{4\}, \{1,2,4\}, \{3,4\}, \{1,2,3,4\}\}$ be a supra topology on X. Then $\{4,5\}$ and $\{1,2,5\}$ are supra $\alpha-$ closed sets but their union $\{1,2,4,5\}$ is a not a supra $\alpha-$ closed set as its complement $\{3\}$ is not supra $\alpha-$ open set .

Definition 2.10.[2]. A function $f:(X,\mu) \to (Y,\mu^*)$ is called a supra $\alpha-$ continuous function if the inverse image of each supra open set in Y is a supra $\alpha-$ open set in X .

3. Types of Supra $\alpha-$ Continuity

Definition 3.1 A function $f:(X,\mu) \to (Y,\mu^*)$ is called $i-$ supra $\alpha-$ continuous if $f^{-1}(V)$ of each

4. Supra α– Compactness

In this section, we present the concept of supra α–compactness and its properties .

Definition 4.1 A collection $\{ A_i : i \in I \}$ of supra α–open sets in a supra topological space (X,μ) is called a supra α– open cover of a subset B of X if $B \subseteq \cup \{ A_i : i \in I \}$ holds .

Definition 4.2 A supra topological space (X,μ) is called supra α– compact if every supra α– open cover of X has a finite sub cover .

Definition 4.3 A subset B of a supra topological space (X,μ) is said to be supra α– compact relative to (X,μ) if, for every collection $\{ A_i : i \in I \}$ of supra α– open subsets of X such that $B \subseteq \cup \{ A_i : i \in I \}$ there exists a finite subset I_o of I such that $B \subseteq \cup \{ A_i : i \in I_o \}$.

Definition 4.4 A subset B of a supra topological space (X,μ) is said to be supra α– compact if B is supra α–compact as a subspace of X.

Theorem 4.5 Every supra α–compact space (X,μ) is supra compact .

Proof : Let $\{ A_i : i \in I \}$ be a supra open cover of X. Since every supra open set in X is a supra α– open set in X. So $\{ A_i : i \in I \}$ is supra α– open cover of (X,μ). Since (X,μ) is a supra α– compact. Therefore the supra α– open cover $\{ A_i : i \in I \}$ of (X,μ) has a finite sub cover say $\{ A_i : i = 1,2,\ldots,n\}$ for X. Hence (X,μ) is a supra compact space .

Theorem 4.6 Every supra α– closed subset of a supra α– compact space is supra α– compact with respect to X .

Proof : Let A be a supra α– closed subset of supra topological space (X,μ). Then $A^c = X - A$ is supra α– open in (X,μ). Let $S = \{ A_i : i \in I \}$ be a supra α– open cover of A by supra α– open subsets in (X,μ). Let $S^* = \{A_i : i \in I \} \cup \{A^c\}$ be a supra α– open cover of (X,μ). That is $X = \cup S^* = (\cup \{A_i : i \in I \}) \cup A^c$. By hypothesis (X,μ) is supra α– compact and hence S^* is reducible to a finite sub cover of (X,μ) say $X = A_1 \cup A_2 \cup \ldots \cup An \cup A^c$; But A and A^c are disjoint. Hence $A \subseteq A_1 \cup A_2 \cup \ldots \cup An$; $A_i \in S$. Thus a supra α– open cover S of A contains a finite sub cover. Hence A is supra α– compact relative to (X,μ) .

Theorem 4.7 A supra α– continuous image of a supra α–compact space is supra compact .

Proof : Let $f: (X,\mu) \to (Y,\mu^*)$ be a supra α–continuous map from a supra α– compact space (X,μ) onto a supra topological space (Y,μ^*) . Let $\{ A_i : i \in I \}$ be a supra open cover of (Y,μ^*) . Then $\{ f^{-1}(A_i) : i \in I \}$ is a supra α– open cover of (X,μ), as f is supra α– continuous. Since (X,μ) is supra α– compact, the supra α– open cover of (X,μ), $\{f^{-1}(A_i) : i \in I \}$ has a finite sub cover say $\{ f^{-1}(A_i) : i = 1,2,\ldots,n$. Therefore $X = \cup \{f^{-1}(A_i) : i = 1,2,\ldots,n\}$, which implies $f(X) = \cup \{ A_i : i = 1,2,\ldots,n\}$, then $Y = \cup \{ A_i : i = 1,2,\ldots,n\}$. That is $\{ A_i : i = 1,2,\ldots,n\}$ is a finite sub

$gof: (X,\mu) \to (Z , \mu^{**})$ is strongly supra α– continuous function .

Proof : Let V be supra α– open set in (Z , μ^{**}) . Since g is strongly α– continuous, $g^{-1}(V)$ is supra open in (Y,μ^*) . We know that every supra open set is supra α – open set, $g^{-1}(V)$ is supra α – open in (Y,μ^*) . Since f is strongly α – continuous, $f^{-1}(g^{-1}(V))$ is supra open in (X,μ), implies $(gof)(V)$ is supra open in (X,μ) . Therefore gof is strongly α – continuous .

Example 3.10 Let $X = Y = Z = \{a,b,c\}$, $\mu = \{\varphi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,c\}\}$, $\mu^* = \{\varphi, Y, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$, $\mu^{**} = \{\varphi, Z, \{a,b\}, \{b,c\}\}$, $f: (X,\mu) \to (Y , \mu^*)$ be the function defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. $g: (Y,\mu^*) \to (Z , \mu^{**})$ be a function defined by $g(a) = c$, $g(b) = b$, $g(c) =$a. Here f and g are strongly supra α – continuous and gof is supra open in (X,μ). Then gof is strongly supra α – continuous function .

Theorem 3.11 If a function $f: (X,\mu) \to (Y,\mu^*)$ a strongly supra α – continuous then it is i– supra α–continuous but not conversely .

Proof : Let $f: (X,\mu) \to (Y,\mu^*)$ be strongly supra α – continuous function. Let G be a supra α – open set in (Y,μ^*). Since f is strongly supra α – continuous, $f^{-1}(G)$ is supra open in (X,μ). Since every supra open set is supra α – open. So every supra closed set is supra α – closed set, $f^{-1}(G)$ is α – open in (X,μ). Hence f is i– supra α– continuous .

The converse of the above theorem need not be true as seen from the following example .

Example 3.12 Let $X = \{a,b,c\}$, $Y = \{1,2,3\}$, $\mu = \{\varphi, X, \{a\}\}$ and $\mu^* = \{\varphi, Y, \{1\}, \{1,2\}\}$. Let $f: (X,\mu) \to (Y,\mu^*)$ be a function defined by $f(a) = 1$, $f(b) = 2$ *and* $f(c) = 3$. Then f is i– supra α–continuous but not strongly supra α– continuous, since for the supra α– open set $\{1,2\}$ in Y . $f^{-1}(\{1,2\}) = \{a,b\}$ is not supra open in X .

Theorem 3.13 If a function $f: (X,\mu) \to (Y,\mu^*)$ is perfectly supra α – continuous then it is i– supra α–continuous but not conversely .

Proof : Let $f: (X,\mu) \to (Y,\mu^*)$ be perfectly supra α – continuous function. Let G be a supra α – open set in (Y , μ^*) . Since f is perfectly supra α – continuous, $f^{-1}(G)$ is both supra open and supra closed in (X,μ) ."Since every supra open set is supra α – open set and so $f^{-1}(G)$ is supra α – open in (X,μ). Hence f is i– supra α– continuous .

The converse of the above theorem need not be true as seen from the following example .

Example 3.14 Let $X = \{a,b,c\}$, $Y = \{1,2,3\}$, $\mu = \{\varphi, X, \{a\}\}$ and $\mu^* = \{\varphi, Y, \{1\}, \{1,2\}\}$. Let $f: (X,\mu) \to (Y,\mu^*)$ be a function defined by $f(a) = 1$, $f(b)2$, and $f(c) = 3$. Then f is i– supra α–continuous but not perfectly supra α – continuous, since for the supra α– open set $\{1,2\}$ in Y, $f^{-1}(\{1,2\}) = \{a,b\}$ is not both supra open and supra closed in X .

$\{(X - A_i) : i \in I\}$ of X has a finite sub cover say $\{(X - A_i) : i = 1,2,...,n\}$. This implies that $X = \cup \{(X - A_i) : i = 1,2,\dots,n\}$, which implies $X = X - I\{A_i : i = 1,2,\dots,n\}$, which implies $X - X = I\{A_i : i = 1,2,\dots,n\}$, and which implies $\varphi = I\{A_i : i = 1,2,\dots,n\}$. This disproves the assumption. Hence $I\{A_i : i \in I\} \neq \varphi$.

Conversely, suppose (X,μ) is not supra α – compact. Then there exist a supra α – open cover of (X,μ) say $\{G_i : i \in I\}$ having no finite sub cover. This implies that for any finite subfamily $\{G_i : i = 1,2,\dots,n\}$ of $\{G_i : i \in I\}$, we have $\cup\{G_i : i = 1,2,\dots,n\} \neq X$, which implies $X - (\cup\{G_i : i = 1,2,\dots,n\}) \neq X - X$, hence $I\{X - G_i : i = 1,2,\dots,n\} \neq \varphi$. Therefore the family $\{X - G_i : i \in I\}$ of supra α – closed sets has a finite intersection property. Then by assumption $I\{X - G_i : i \in I\} \neq \varphi$, which implies $X - (\cup\{G_i : i \in I\}) \neq \varphi$, so that $\cup\{G_i : i \in I\} \neq X$. This implies that $\{G_i : i \in I\}$ is not a cover of (X,μ). This disproves the fact that $\{G_i : i \in I\}$ is a cover for (X,μ). Therefore any supra α – open cover $\{G_i : i \in I\}$ of (X,μ) has a finite sub cover $\{G_i : i = 1,2,\dots,n\}$. Hence (X,μ) is supra α – compact.

Theorem 4.12 Let $f:(X,\mu) \to (Y,\mu^*)$ is i– supra α– continuous and a subset B of X is supra α – compact relative to X. Then $f(B)$ is supra α – compact relative to Y.

Proof : Let $\{A_i : i \in I\}$ be a cover of $f(B)$ by supra α – open subsets of Y. Since f is i– supra α– continuous. Then $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by supra α – open subsets of X. Since B is supra α – compact relative to X, so $\{f^{-1}(A_i) : i \in I\}$ has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(An)\}$ for B. Then it implies that $\{A_i : i = 1,2,\dots,n\}$ is a finite subcover of $\{A_i : i \in I\}$ for $f(B)$. So $f(B)$ is supra α – compact relative to Y.

5. Countably Supra α – Compactness

In this section, we present the concept of countably supra α – compactness and its properties .

Definition 5.1.[8] A supra topological space (X,μ) is said to be countably supra compact if every countable supra open cover of (X,μ) has a finite subcover .

Definition 5.2 A supra topological space (X,μ) is said to be countably supra α – compact if every countable supra α – open cover of X has a finite subcover .

Theorem 5.3 If (X,μ) is a countably supra α – compact space, then (X,μ) is countably supra compact .

Proof : Let (X,μ) be a countubly supra α – compact . Let $\{A_i : i \in I\}$ be a countable supra open cover of (X,μ) . Since every supra open set in X is always supra α – open set in X . So $\{A_i : i \in I\}$ is a countable supra α – open cover of (X,μ) . Since (X,μ) is countable supra α – compact, so the countable supra α – open cover $\{A_i : i \in I\}$ cover of $\{A_i : i \in I\}$ for (Y,μ^*). Hence (Y,μ^*) is supra compact .

Theorem 4.8 Let $f:(X,\mu) \to (Y,\mu^*)$ is strongly supra α– continuous map from a supra compact space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is supra α– compact .

Proof : Let $\{A_i : i \in I\}$ be a supra α– open cover of (Y,μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra open cover of (X,μ) , since f is strongly supra α– continuous . Since (X,μ) is supra compact, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,\dots,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,\dots,n\}$, which implies $f(X) = \cup\{A_i : i = 1,2,\dots,n\}$, so that $Y = \cup\{A_i : i = 1,2,\dots,n\}$. That is $\{A_1, A_2, \dots, An\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,μ^*) . Hence (Y,μ^*) is supra α – compact .

Theorem 4.9 Let $f:(X,\mu) \to (Y,\mu^*)$ is perfectly supra α– continuous map from a supra compact space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is supra α – compact .

Proof : Let $\{A_i : i \in I\}$ be a supra α – open cover of (Y,μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra open cover of (X,μ), since f is perfectly supra α – continuous . Since (X,μ) is supra compact, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a finite subcover say $\{f^{-1}(A_i) : i = 1,2,\dots,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,..,n\}$, which implies $f(X) = \cup\{A_i : i = 1,2,..,n\}$, so that $Y = \cup\{A_i : i = 1,2,\dots,n\}$. That is $\{A_1, A_2, \dots, An\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y,μ^*). Hence (Y,μ^*) is supra α – compact .

Theorem 4.10 Let $f:(X,\mu) \to (Y,\mu^*)$ is i– supra α– continuous from a supra α – compact space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is supra α – compact .

Proof : Let $\{A_i : i \in I\}$ be a supra α – open cover of (Y,μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra α – open cover of (X,μ), since f is i– supra α– continuous . Since (X,μ) is supra α – compact, the supra α – open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,\dots,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,\dots,n\}$, which implies $f(X) = \cup\{A_i : i = 1,2,\dots,n\}$, so that $Y = \cup\{A_i : i = 1,2,\dots,n\}$. That is $\{A_1, A_2, \dots, An\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y,μ^*) . Hence (Y,μ^*) is supra α – compact .

Theorem 4.11 A supra topological space (X,μ) is supra α – compact if and only if every family of supra α – closed sets of (X,μ) having finite intersection property has a non-empty intersection.

Proof : Suppose (X,μ) is supra α – compact, Let $\{A_i : i \in I\}$ be a family of supra α – closed sets with finite intersection property . Suppose $I_{i \in I} A_i = \varphi$, then $X - I(\{A_i : i \in I\}) = X$. This implies $\cup\{(X - A_i) : i \in I\} = X$. Thus $\{(X - A_i) : i \in I\}$ is a supra α – open cover of (X,μ) . Then as (X,μ) is supra α – compact, the supra α – open cover compact space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is countably supra $\alpha-$compact .

Proof : Let $\{ A_i : i \in I\}$ be a countable supra $\alpha-$open cover of (Y,μ^*). Since f is perfectly supra $\alpha-$continuous map, $\{f^{-1}(A_i) : i \in I\}$ is countable supra open cover and countable supra closed cover of (X,μ). Again since (X,μ) is countably supra compact space the countable supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a finite subcover $\{f^{-1}(A_i) : i = 1,2,\dots,n\}$. Therefore $X = \cup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X) = \cup_{i=1}^{n} (A_i)$. Then $Y = \cup_{i=1}^{n} (A_i)$ is finite sub cover of $\{ A_i : i \in I\}$ for (Y,μ^*). Hence (Y,μ^*) is countably supra $\alpha-$compact .

Theorem 5.9 If $f:(X,\mu) \to (Y,\mu^*)$ is $i-$ supra $\alpha-$continuous from a countably supra $\alpha-$compact space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is countably supra $\alpha-$compact .

Proof : Let $\{ A_i : i \in I\}$ be a countable supra $\alpha-$open cover of (Y,μ^*) . Since f is $i-$ supra $\alpha-$continuous, $\{f^{-1}(A_i) : i \in I\}$ is countable supra $\alpha-$open cover of (X,μ) . Again since (X,μ) is countably supra $\alpha-$compact space, the countable supra $\alpha-$open cover $\{f^{-1}(A_i) : i \in I\}$of (X,μ) has a finite subcover $\{f^{-1}(A_i) : i = 1,2,\dots,n\}$. Therefore $X = \cup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X) = \cup_{i=1}^{n} (A_i)$. Then $Y = \cup_{i=1}^{n} (A_i)$ is finite sub cover of $\{ A_i : i \in I\}$for (Y,μ^*). Hence (Y,μ^*) is countably supra $\alpha-$compact .

6. Supra $\alpha-$Lindelof Space

In this section, we concentrate on the concept of supra $\alpha-$Lindelof space and its properties .

Definition 6.1. [8] A supra topological space (X,μ) is said to be supra Lindelof space if every supra open covre of (X,μ) has a countable sub cover .

Definition 6.2 A supra topological space (X,μ) is said to be supra $\alpha-$Lindelof space if every supra $\alpha-$open cover of (X,μ) has a countable sub cover .

Theorem 6.3. Every supra $\alpha-$Lindelof space (X,μ) is supra Lindelof space .

Proof : Let (X,μ) be a supra $\alpha-$Lindelof space . Let $\{ A_i : i \in I\}$ be a supra open cover of (X,μ). Since every supra open set in X is always supra $\alpha-$open set in X .Therefore $\{ A_i : i \in I\}$ is supra $\alpha-$open cover of (X,μ). Since (X,μ) is supra $\alpha-$Lindelof, so the supra $\alpha-$open cover $\{ A_i : i \in I\}$of (X,μ) has a countable subcover say $\{ A_i : i = 1,2,\dots,n\}$ for X. Hence (X,μ) is a supra Lindelof space .

Theorem 6.4. If (X,μ) is supra $\alpha-$Lindelof space, then $(X,\mathcal{T})$ is Lindelof space .

Proof : Let $\{ A_i : i \in I\}$ be an open cover of X . Since every open set in X being a supra open set in X is also supra $\alpha-$open set in X . Therefore $\{ A_i : i \in I\}$ is a supra $\alpha-$open cover of (X,μ) . Since (X,μ) is supra $\alpha-$Lindelof, so the supra $\alpha-$open cover $\{ A_i : i \in I\}$ of (X,μ) has a countable subcover say $\{ A_i : i = 1,2,\dots,n\}$ for X. Hence $(X,\mathcal{T})$ is a Lindelof space .

has a finite sub cover say $\{ A_i : i = 1,2,\dots,n\}$ for X . Hence (X,μ) is a countably supra compact space.

Theorem 5.4 If (X,μ) is countably supra compact and every supra $\alpha-$closed subset of X is supra closed in X, then (X,μ) is countably supra $\alpha-$compact .

Proof : Let (X,μ) be a countably supra compact space . Let $\{ A_i : i \in I\}$ be a countable supra $\alpha-$open cover of (X,μ) . since every supra $\alpha-$closed subset of X is supra closed in X . Thus every supra $\alpha-$open set in X is supra open in X . Therefore $\{ A_i : i \in I\}$ is a countable supra open cover of (X,μ). Since (X,μ) is countably supra compact . So the countable supra open cover $\{ A_i : i \in I\}$ of (X,μ) has a finite sub cover say $\{ A_i : i = 1,2,\dots,n\}$ for X .Hence (X,μ) is a countably supra $\alpha-$compact space .

Theorem 5.5. Every supra $\alpha-$compact space is countably supra $\alpha-$compact .

Proof : Let (X,μ) be supra $\alpha-$compact space . Let $\{ A_i : i \in I\}$ be a countable supra $\alpha-$open cover of (X,μ) . Since (X,μ) is supra $\alpha-$compact, so the supra $\alpha-$open cover $\{ A_i : i \in I\}$ of (X,μ) has a finite sub cover say $\{ A_i : i = 1,2,\dots,n\}$ for (X,μ) . Hence (X,μ) is countably supra $\alpha-$compact space .

Theorem 5.6 If $f:(X,\mu) \to (Y,\mu^*)$ is supra $\alpha-$continuous map from a countably supra $\alpha-$compact space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is countably supra compact .

Proof : Let $\{ A_i : i \in I\}$ be a countable supra open cover of (Y,μ^*). Since f is supra $\alpha-$continuous map, $\{f^{-1}(A_i) : i \in I\}$ is countable supra $\alpha-$open cover of (X,μ). Again since (X,μ) is countably supra $\alpha-$compact space, then $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a finite subcover $\{f^{-1}(A_i) : i = 1,2,\dots,n\}$. Therefore $X = \cup_{i=1}^{n} f^{-1}(A_i)$. which implies $f(X) = \cup_{i=1}^{n} (A_i)$. Then $Y = \cup_{i=1}^{n} (A_i)$ is finite subcover of $\{ A_i : i \in I\}$ for (Y,μ^*). (Y,μ^*) is countably supra compact .

Theorem 5.7 If $f:(X,\mu) \to (Y,\mu^*)$ is strongly supra supra $\alpha-$continuous map from a countably supra compact space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is countable supra $\alpha-$compact .

Proof : Let $\{ A_i : i \in I\}$ be a countable supra $\alpha-$open cover of (Y,μ^*). Since f is strongly supra $\alpha-$continuous map, $\{f^{-1}(A_i) : i \in I\}$ is countable supra open cover of (X,μ). Again since (X,μ) is countable supra compact space, the countable supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a finite subcover $\{f^{-1}(A_i) : i = 1,2,\dots,n\}$. Therefore $X = \cup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X) = \cup_{i=1}^{n} (A_i)$. Then $Y = \cup_{i=1}^{n} (A_i)$ is finite subcover of $\{ A_i : i \in I\}$ for (Y,μ^*). Hence (Y,μ^*) is countably supra $\alpha-$compact .

Theorem 5.8 If $f:(X,\mu) \to (Y,\mu^*)$ is perfectly supra $\alpha-$continuous map from a countably supra

$\alpha-$ Lindelof space, the supra $\alpha-$ open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a countable subcover $\{f^{-1}(A_i) : i = 1,2,\ldots,$. Therefore $X = \cup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X) = \cup_{i=1}^{n} (A_i)$. Then $Y = \cup_{i=1}^{n} (A_i)$. is a countable sub cover of $\{A_i : i \in I\}$ for (Y,μ^*). Hence (Y,μ^*) is supra $\alpha-$ Lindeiof .

Theorem 6.9 If $f:(X,\mu) \to (Y,\mu^*)$ is perfectly supra $\alpha-$ continuous map from a supra Lindelof space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is supra $\alpha-$ Lindelof .

Proof : Let $\{A_i : i \in I\}$ be a supra $\alpha-$ open cover of (Y,μ^*) . Since f is perfectly supra $\alpha-$ continuous map, $\{f^{-1}(A_i) : i \in I\}$ is supra open cover and supra closed cover of (X,μ) . Again since (X,μ) is supra Lindelof space, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a countable sub cover $\{f^{-1}(A_i) : i = 1,2,\ldots,n$. Therefore $X = \cup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X) = \cup_{i=1}^{n} (A_i)$. Then $Y = \cup_{i=1}^{n} (A_i)$. is a countable sub cover of $\{A_i : i \in I\}$ for (Y,μ^*). Hence (Y,μ^*) is supra $\alpha-$ Lindelof .

Theorem 6.10 If (X,μ) is supra $\alpha-$ Lindelof space and countably supra $\alpha-$ compact space, then (X,μ) is supra $\alpha-$ compact space .

Proof: Suppose (X,μ) is supra $\alpha-$ Lindelof space and countably supra $\alpha-$ compact space . Let $\{A_i : i \in I\}$ be a supra $\alpha-$ open cover of (X,μ) . Since (X,μ) is supra $\alpha-$ Lindelof space, $\{A_i : i \in I\}$ has a countable sub cover say $\{A_i : i \in I_o\}$ for some $I_o \subseteq I$ and I_o is countable . Therefore $\{A_i : i \in I_o\}$ is a countable supra $\alpha-$ open cover of (X,μ) . Again, since (X,μ) is countably supra $\alpha-$ compact space $\{A_i : i \in I_o\}$ has a finite sub cover and say $\{A_i : i = 1,2,\ldots,n\}$. Therefore $\{A_i : i = 1,2,\ldots,n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (X,μ) . Hence (X,μ) is a supra $\alpha-$ compact space .

Theorem 6.11. If $f:(X,\mu) \to (Y,\mu^*)$ is $i-$ supra $\alpha-$ continuous and a subset B of X is supra $\alpha-$ Lindelof relative to X, then $f(B)$ is supra $\alpha-$ Lindelof relative to Y .

Proof : Let $\{A_i : i \in I\}$ be a cover of $f(B)$ by supra $\alpha-$ open subsets of Y. By hypothesis f is supra $\alpha-$ irresolute and so $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by supra $\alpha-$ open subsets of X. Since B is supra $\alpha-$ Lindelof relative to X, $\{f^{-1}(A_i) : i \in I\}$ has a countable subcover say $\{f^{-1}(A_i) : i \in I_o\}$ for B, where I_o is a countable subset of I. Now $\{A_i : i \in I_o\}$ is a countable subcover of $\{A_i : i \in I\}$ for $f(B)$. So $f(B)$ is supra $\alpha-$ Lindelof relative to Y .

Theorem 6.5. Every supra $\alpha-$ compact space is supra $\alpha-$ Lindelof space .

Proof : Let (X,μ) be a supra $\alpha-$ compact space. Let $\{A_i : i \in I\}$ be a supra $\alpha-$ open cover of (X,μ) . Since (X,μ) is supra $\alpha-$ compact space . Then $\{A_i : i \in I\}$ has a finite sub cover say $\{A_i : i = 1,2,\ldots,n\}$. Since every finite sub cover is always countable sub cover and therefore $\{A_i : i = 1,2,\ldots,n\}$ is a countable sub cover of $\{A_i : i \in I\}$. Hence (X,μ) is a supra $\alpha-$ Lindelof space .

Theorem 6.6 If $f:(X,\mu) \to (Y,\mu^*)$ is supra $\alpha-$ continuous map from a supra $\alpha-$ Lindelof space (X,μ) onto a supra topological space (Y,μ^*) then (Y,μ^*) is supra Lindelof space .

Proof : Let $\{A_i : i \in I\}$be a supra open cover of (Y,μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra $\alpha-$ open cover of (X,μ), as f is supra $\alpha-$ continuous . Since (X,μ) is supra $\alpha-$ Lindelof space, so the supra $\alpha-$ open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a countable sub cover say $\{f^{-1}(A_i) : i \in I_o\}$ for some $I_o \subseteq I$ and I_o is countable. Therefore $X = \cup\{f^{-1}(A_i) : i \in I_o\}$, which implies $f(X) = \cup\{A_i : i \in I_o\}$, then $Y = \cup\{A_i : i \in I_o\}$. That is $\{A_i : i \in I_o\}$ is a countable sub cover of $\{A_i : i \in I\}$ for (Y,μ^*) . Hence (Y,μ^*) is a supra Lindelof space.

Theorem 6.7 If $f:(X,\mu) \to (Y,\mu^*)$ is $i-$ supra $\alpha-$ continuous from a supra $\alpha-$ Lindelf space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is supra $\alpha-$ Lindelof space .

Proof : Let $\{A_i : i \in I\}$ be a supra $\alpha-$ open cover of (Y,μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra $\alpha-$ open cover of (X,μ) . Since f is $i-$ supra $\alpha-$ continuous . As (X,μ) is supra $\alpha-$ Lindelof space, so the supra $\alpha-$ open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,μ) has a countable sub cover say $\{f^{-1}(A_i) : i \in I_o\}$ for some $I_o \subseteq I$ and I_o is countable. Therefore $X = \cup\{f^{-1}(A_i) : i \in I_o\}$, which implies $f(X) = \cup\{A_i : i \in I_o\}$, so that $Y = \cup\{A_i : i \in I_o\}$. That is $\{A_i : i \in I_o\}$ is a countable sub cover of $\{A_i : i \in I\}$ for (Y,μ^*) . Hence (Y,μ^*) is a supra $\alpha-$ Lindelof space .

Theorem 6.8 If $f:(X,\mu) \to (Y,\mu^*)$ is strongly supra $\alpha-$ continuous map from a supra $\alpha-$ Lindelof space (X,μ) onto a supra topological space (Y,μ^*), then (Y,μ^*) is supra $\alpha-$ Lindelof .

Proof : Let $\{A_i : i \in I\}$ be a supra $\alpha-$ open cover of (Y,μ^*) . Since f is strongly supra $\alpha-$ continuous map, $\{f^{-1}(A_i) : i \in I\}$ is supra open cover of (X,μ), which implies $\{f^{-1}(A_i) : i \in I\}$ is supra $\alpha-$ open cover of (X,μ) . Again since (X,μ) is supra

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حول التراص الفوقي من النمط α في الفضاءات التبولوجية الفوقية

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الملخص

الغرض من البحث هو تقديم مفهوم الدالة المستمرة الفوقية بقوة من النمط α والدالة المستمرة الفوقية التامة من النمط α والدالة المستمرة الفوقية الكلية من النمط α. قد درست العلاقات بين هذه الدوال وتحرينا بعض خواصها. وكذلك قدمنا مفهوم الفضاء المرصوص الفوقي من النمط α والفضاء الندلوفي الفوقي من النمط α والفضاءات المرصوصة المعدودة الفوقية من النمط α وبعض الخصائص الأساسية قد برهنت. وأخيرا, العلاقات بين الدوال المفتوحة الفوقية من النمط α والدوال المستمرة الفوقية من النمط α والدوال المحيرة الفوقية من النمط α في الفضاءات التبولوجية الفوقية قد درست.