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WE-primary submodules and WE-quasi-prime submodules

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ABSTRACT

In this paper we introduced and study two concepts one of is a subclass of a class of weakly primary submodules called WE-primary submodules and the other of a subclass of weakly quasi-prime submodules called WE-quasi-prime submodules. Several properties of WE-primary and WE- quasi - prime submodules have been given, and some characterizations of each type of submodules are introduced moreover behavior of these submodules in some classes of modules are investigated. On the otherhand, the homeorphic and inverse image of these concept under certain conditions are proved.

1- Introduction

Weakly primary and weakly quasi-prime submodules and introduced by Ebahrmi in 2005 and by AL-joboury in 2013, respectively, where a porper submodules V of an R-modules X is called a weakly primary, if where her 0≠r x ∈ V, where rϵR , x ∈ X , implies that either x∈V or $r^nX \subseteq V$ for some positive integer n [1], and a proper submodule W of an R-module X is called a weakly quasi-prime ,if wherever 0≠rsx∈ W, where $r, s \in R, x \in X,$ *implies that either* $rx \in W$ or sx∈ W[2] .

Throughout that paper all rings will be commutative with identity and all R-modules are unitary. An R-module X is called multiplication if every submodule N of X is of the from N=IX for some ideal I of R [3]. Recall that an R-module X is called duo module , if every submodules N of X is fully invariant where a submodule N of X is called fully invariant if f(N) ⊆ N for each f ∈ End (X) [4]. Recall that an element x∈ X called torsion if 0 ≠ ann (x)={r∈ R:rx=0. The set of all torsion element denoted by τ(X), which is a submodule of X if τ(X) =(0) , the R-module X is called torsion free [5]. Recall that an R-module X is called scalar module if for each f ∈End(X) ,∃ r∈ R such that f(x) =rx for each x∈X [6].

2- Basic properly of WE-primary submodules

The notion of WE-primary submodules are introduce in this part of this note, as a subclass of the class of a weakly primary submodules. We investigate some basic properties of WE-primary submodules and give some characterizations of its.

Definition 2.1 : A proper submodule V of an R-module X is said to be WE-primary, if wherever 0≠α(x) ∈ V where α ∈End(X), x ∈ X implies that either x ∈ V or $\alpha^n(X) \subseteq V$ for some positive integer n.

And an idea J of a ring R is called a WE-primary if I is a WE- primary as an R-submodule of an R-module R .

Remark 2.2 : Every WE-primary submodule of an R-module X is a weakly primary, while the converse is not true ingeneral.

Proof: let B be a WE-primary submodule of X and 0≠cx ∈ V, where c ∈ R,
x∈X, with x∉ V. Let α : X ⟶X define by α(x)=cx for all x ∈ X
α ∈ End(X). 0 ≠ cx = α(x) ∈ V, implies that $\alpha^n(X) \subseteq V$ for some positive integer n. It follows that $c^nX \subseteq V$.
For the converse consider the following example:
Let X= $Z_{25} \oplus Z$, R=Z , V= $(\bar{0}) \oplus 25Z$.
Clearly V is a weakly primary submodule of X, but not WE-primary, since if

$\alpha : X \longrightarrow X$ defined by $\alpha(\bar{x}, y) = (\bar{0}, y)$.
for all $\bar{x} \in Z_{25}$, $y \in Z$, $\alpha \in End(X)$, and $0 \neq \alpha(\bar{1}, 25) = (\bar{0}, 25) \in V$, but $(\bar{1}, 25) \notin V$ and $\alpha^n(Z_{25} \oplus Z) = (\bar{0}) \oplus Z \nsubseteq V$ for some positive integer n.

The following proposition shows that WE-primary submodules and weakly primary submodule are equivalent in the class of multiplication modules.

Proposition 2.3 : Let X be a multiplication R-module , and V a proper submodule of X . Then V is weakly primary submodule if and only if V is a WE-primary submodule .

Proof: Let $0 \neq \alpha(x) \in V$, where $\alpha \in End(X)$, $x \in X$ and $x \notin V$. Since X is a multiplication , then by [3,lemma 1.4]. There exists r∈R such that $\alpha(x) = rx$ *for each* $x \in X$. Thus $0 \neq \alpha(x) = rx \in V$, and $x \notin V$, implies that $r^nX \subseteq V$ for some positive integer n . That is $\alpha^n(X) \subseteq V$. Hence V is a WE-primary submodule .

(⇐)Direct .

It is well known that cyclic R-modules are multiplication we get the following result.

Corollary 2.4 : Let X be a cyclic R-module, and V be a proper submodule of X, then V is weakly primary submodule if and only if V is WE-primary submodule.

Proposition 2.5 : Let X be an R-module, and V be a WE-primary submodule of X, then for each $x \in X$ and $x \notin V$ $\sqrt{[V:Rx]} = \sqrt{[V:X]} \cup [(0):Rx]$.

Proof Since V is a WE-primary submodule of X then V is a weakly primary submodule.

Now, let $c \in \sqrt{[V:Rx]}$, implies that $c^n \in [V:Rx]$ for some positive integer n, then $c^nx \in V$. If $0 \neq c^nx \in V$, and V is weakly primary and $x \notin V$, then $c^nX \subseteq V$, implies that $c^n \in [V:X]$ and hence $c \in \sqrt{[V:X]}$. If $0 = c^nx$ then suppose that m is the smallest integer with $c^mx=0$. If m=1 then $c \in [(0):Rx]$, otherwise $c \in \sqrt{[V:X]}$, so

$\sqrt{[V:Rx]} \subseteq \sqrt{[V:X]} \cup [(0):Rx]$.

Now, let $c_1 \in \sqrt{[V:X]} \cup [(0):Rx]$, implies that $c_1 \in \sqrt{[V:X]}$ or $c_1 \in [(0):Rx]$.

If $c_1 \in [(0):Rx] \subseteq [V:Rx]$ implies that $c_1 \in [V:Rx] \subseteq \sqrt{[V:Rx]}$.

If $c_1 \in \sqrt{[V:X]}$ implies that $c_1^{n1} \in [V:X] \subseteq [V:Rx]$

For some positive integer n_1, so $c_1 \in \sqrt{[V:Rx]}$. Hence in both case $c_1 \in \sqrt{[V:Rx]}$. Hence $\sqrt{[V:X]} \cup [(0):Rx] \subseteq \sqrt{[V:Rx]}$ .Thus $\sqrt{[V:Rx]} = \sqrt{[V:X]} \cup [(0):Rx]$.

Proposition 2:6 : Let X be a faithful cyclic R-module and V be a WE-primary submodule of Y, then [V:X] is a WE-primary ideal of R.

Proof : Since V be a WE-primary submodule of X, then V a weakly primary submodule of X . Then by [7;prop.2.2] [V:X] is a weakly primary ideal of R. But R is cyclic, then by corollary 2.4 [V:X] is a WE-primary ideal of R.

Recall that a proper submodule V of R-module X is called WE-prime if whenever $0 \neq \alpha(x) \in V$, with $\alpha \in End(X)$ and $x \in X$, implies that either $x \in V$ or $\alpha(X) \subseteq V$, and an ideal I of a ring R is called WE-prime , if I is a WE-prime R-submodule of an R-module R[8] .

Proposition 2:7 : Let X be a faithful cyclic R-module and V be a WE-primary submodule , then $\sqrt{[V:X]}$ is a WE-prime idea of R.

Proof : Since V is a WE-primary submodule of X then V is weakly primary submodule of X then by [7,prop. 2.3] we get $\sqrt{[V:X]}$ is a weakly prime ideal of R. But R is cyclic, then by [8,prop. 2.4], we get $\sqrt{[V:X]}$ is a WE-prime ideal of R.

Proposition 2.8 : Let X be a torsion free cyclic R-module over an integer domain R, and V be a WE-primary submodule of X . Then [V:X] is a WE-primary idea of R.

Proof: Since V is a WE-primary submodule of X then V is a weakly primary submodule of X. Since X is torsion free over integral domain, then by [9,prop. 3.3] we get [V:X] is a weakly primary idea lot R. Thus by corollary 2.4 [V:X] is a WE-primary idea of R.

Proposition 2.9 : Let $f : X \longrightarrow X'$ be an R-epimeorphism , and V be a proper submodule of X , with ker f ⊆ V, if V is a WE-primary submodule of X, then f (V) is a WE-primary submodule of X', where X' is X-projective module.

Proof : Clearly f(V) is a proper submodule of X'. Let $0 \neq \alpha(x') \in f(V)$, where $x' \in X'$, $\alpha \in End(X')$, and $x' \notin f(V)$. Since f is an epimorphism, then ,∃ $x \in X$ such that $f(x) = x' \notin f(V)$, it follows that $x \notin V$. Since X' is X-projective, then there exist $\beta : X \longrightarrow X'$ such that for $fo\beta = \alpha$. Thus $0 \neq \alpha(x') \in f(V)$, implies that $0 \neq (fo\beta)(x') \in f(V)$, it follows that $0 \neq (fo\beta)(f(x)) \in f(V)$ then there exist a nonzero element $v \in V$ such that $(fo\beta)(f(x)) = f(V)$ it follows that $0 \neq \beta$ of $(x) \in$ kerf ⊆ V so, $0 \neq \beta of(x) \in V$. But V is a WE-primary submodule of X and $x \notin V$,then $(\beta of)^n(X) \subseteq V$. Thus $f^n(\beta of)^n(X) \subseteq f^n(V) \subseteq f(V)$, hence $(fo\beta)^n f^n(X) \subseteq f(V)$ implies that $\alpha^n(X') \subseteq f(V)$. As a direct application of proposition 2.9 , we get the following result.

Corollary 2.10 : Let V be WE-primary submodule of an R-module X, then for any submodule W of X with W⊆ V then V/W is a WE-primary submodule of X/W , wherever X/W is an X- projective module .

Proposition 2.11 : Let f: X→X be R-monomorphism and V be a proper fully invariant WE-primary submodule of X with $f(X) \nsubseteq V$. Then $f^{-1}(V)$ is a WE-primary submodule of X.

Proof : Clearly $f^{-1}(V)$ is a proper submodule of X.
let $0 \neq \alpha(x) \in f^{-1}(V)$, $x \in X$, $\alpha \in End(X)$, and $x \notin f^{-1}(V)$. Then $f(x) \notin V$. Since V is fully invariant submodule then $x \notin V$. Since $0 \neq \alpha(x) \in f^{-1}(V)$, then $0 \neq fo\alpha(x) \in V$. But V is a WE-primary submodule of X, and $x \notin V$ then $(fo\alpha)^n(X) \subseteq V$ for some positive integer n . Since f is monomorphism , then $f^n o \alpha^n(X) \subseteq V$ implien that $\alpha^n(X) \subseteq f^{-n}(V) \subseteq f^{-1}(V)$. Therefore $f^{-1}(V)$ is a WE-primary submodule of X.

proposition 2.12: Let X be an R-module, and V ,W are submodules of X with W is a fully invariant submodule of X and W ⊆ V such that V/W is a WE-primary submodule of X/W. Then V is a WE-primary submodule of X.

proof : let 0 ≠ α(x) ∈ V, where x ∈ X, α ∈End (X) and x ∉ V. let β:X/W →X/W defined by β(x+W)= α(x)+W for all x ∈ X . Clearly β is a well-define since W is a fully invariant submodule of X. Since 0 ≠ α(x)∈V, implies that 0 ≠ f(x)+W= β(x+W)∈ V/W. But V/W is a WE-primary submodule of X/W and x+W∉V/W implies that β^n(X/W) ⊆ V/Wfor some positive integer n. Thus (α(x)+W)/W ⊆ V/W, implies that α^n(X)+W⊆ V , hence α^n(X)⊆ V . Thus V is a WE-primary submodule of X .

3- Basic properties of WE-quasi-prime submodules.

In this section, we introduced the concept of a WE-quasi-prime submodule as a stronger form of the concept of weakly quasi-prime submodule. A number of results concerning WE-quasi-prime submodules are given. Also various basic properties of a WE-quasi-prime submodule are considered.

Definition 3.1: A proper submodule W of an R-module X is said to be a WE-quasi- prime submodule if wherever 0≠(α oβ)(x) ∈ W, with α,β ∈ End(X) , x ∈ X, implies that either α(x) ∈ W or β(y) ∈ W.

An idea I of a ring R is called WE-quasi-prime, if I is a WE-quasi-prime R-submodule of an R-module R.

Remarks and examples 3.2 :

1-Every WE-quasi-prime submodule of an R-module X is a weakly quasi-prime, while the converse is not true ingeneral.

Proof :Let W be a WE-quasi-prime submodule of X, and 0≠cd x ∈ W , where c, d ∈ R , x ∈ X. Let α,β:X→X defined by α(x) =c x, β(x)=d x , for all x ∈ X ,α , β ∈ End(X). Thus 0≠cd x = (α oβ)(x) ∈ W, but W is a WE-quasi-prime, then either α(x) ∈ W or β(x) ∈ W. That is either c x ∈ W or d x ∈ W . Hence W is a weakly quasi-prime submodule of X.

For the converse consider the following example :

Let X= Z⊕Z , R=Z , W=(0)⊕ 5Z. Clearly W is a weakly quasi-prime submodule , but not WE-quasi-prime submodule , since α,β:X→X defined by $\alpha(x_1 , x_2)=(x_2 , x_1)$, $\beta(x_1 , x_2)=(0,x_1)$, α,β ∈ End(X) . Consider $0\neq\alpha_0\beta(1,10)$ = (0,10) ∈ W , but α (1,10) = (10,1) ∉ W and β(1,10)=(0,1) ∉ W .

2-Every WE-prime submodule W of an R-module X is a WE-quasi-prime , but the converse is not true ingeneral.

Proof: Let 0≠α o β(x) ∈ W , x ∈ X ,α , β ∈ End (X),and let α(x) ∉ W.

That is 0≠α (β(x)) ∈ W. Since W is a WE-prime, then either β(x) ∈ W or α(X) ⊆W, but α(x) ∉ W, implies that β(x)∈W.

Hence β(x) ∈ W. Therefore W is a WE-quasi-prime.

For the converse consider the following example. Let X= Z_3⊕Z, R=Z, W=($\bar{0}$)⊕3Z. Clearly W is not WE-prime (see[5,example2.3]). But W is a WE-quasi-prime submodule of X, since α,β:X→X defined by α($\bar{a}$, b) = ($\bar{0}$,b) and β($\bar{a}$, b) = ($\bar{0}$,3b) , $\bar{a}$ ∈ Z_3 , b ∈ Z.

Consider 0≠α o β($\bar{a}$,b)= ($\bar{0}$,3b) ∈ W, implies that eitherα($\bar{a}$, b) ∈ W or β($\bar{a}$, b) ∈ W clearly β($\bar{a}$, b) = ($\bar{0}$,3b) ∈ W.

Proposition 3.3 : Let X be a scalar module and W is a proper submodule of X, then W is a WE-quasi-prime , if and only if W is a weakly quasi-prime.

Proof : (⇒) Direct.

⇐) let 0≠α o β(x) ∈ W, with α , β ∈End (X) , x ∈ X. (Since X is scalar module, then ∃ r,s ∈ R such that α(x) =rx and β(x) =sx for all x ∈ X. Thus 0≠α o β(x)=rsx ∈W. Since W is a weakly quasi-prime submodule of X , then either rx ∈W or sx ∈ W. Hence either α(x) ∈ W or β(x) ∈ W .

Proposition 3.4 : Let X be an R-module , and V be a proper submodule of X . if V is a WE-quasi-prime with 0≠α o β(L) ⊆ V, where α , β ∈ End (X) and L is a submodule of X with L ⊈ V , then either α(L) ⊆ V or β(L) ⊆ V.

Proof: It is given that 0≠α o β(L) . Suppose that α(L) ⊈ V and β(L) ⊈ V ,then their exists. x_1 , x_2∈ L , such that α(x_1) ∉ V and β(x_2)∉ V

Now , 0≠(α o β)(x_1) ∈ V and α(x_1) ∉ V, we get β(x_1)∈ V . Also 0≠(α o β)(x_2) ∈ V and β (x_2) ∉ V .

we get α (x_2) ∈ V . Hence 0≠(α o β)(x_1+x_2) ∈ V either α (x_1+x_2) ∈ V or β(x_1+x_2) ∈ V .

If α (x_1+x_2) ∈ V , implies that α(x_1)+ α(x_2) ∈ V, and since α(x_2) ∈ V which is a contradiction. If β(x_1+x_2) ∈ V implies that β(x_1)+ β(x_2) ∈ V and since β(x_2) ∉ V , we get β(x_1) ∈ V which is a contradiction. Therefore , we get either α(L) ⊆ V or β(L) ⊆ V.

The following theorem given characterizations of a WE-quasi-prime submodule.

Theorem 3.5 :Let X be an R-module, and V be a proper submodule of X. Then the following statements are equivalent :

1-V is a WE- quasi-prime submodule of X .

2-for every α , β ∈ End(X), [V : α o β] = [(0) : α o β] ∪ [V : α] ∪ [V : β].

3-for every α ∈ End(X), and x ∈ X , with α(x) ∉V. [V : α(x)] = [(0) : α (x)] ∪ [V:x]

4- for any α ∈ End (X), and x ∈ X , with α(x) ∉V, [V: α(x)] = [(0) : α (x)] or . [V : α(x)] = [V:x] .

Proof: (1)→(2) let x∈ [V:α o β] implies that α o β (x) ∈ V. If α o β (x) =0 , implies that x ∈ [(0) : α o β]. Assume that 0≠α o β(x) ∈ V . Since V is a WE-quasi–prime then either α(x) ∈ V or β (x) ∈ V. That is either x ∈ [V :α] or x ∈ [V :β]. Consequently,

[V : α o β] = [(0) : α o β] ∪ [V : α] ∪ [V : β] .

(2)→(3) : Suppose that α(x) ∉V where α ∈ End(X) , and x ∈ X , and let

β ∈[V : α(x)], then α o β(x) ∈ V , and so x ∈ [V: α o β] , since α(x) ∉V , then x ∉[V: α] . Thus by (2) x ∈[(0) : α(x)] or x ∈ [V: β] , hence

β ∈[(0) : α(x)] or β ∈ [V:x] . Therefore [V : α(x)] =[(0) : α(x)] ∪[V : x] .

(3)→(4) follows by the fact if an ideal is the union of two ideals , then it is equal to one of them .

(4)→(1) let 0≠α o β(x) ∈ V with α(x) ∉V , implies that β ∈[V : α(x)]
= [V:x] . And β ∉[(0) : α(x)] . Then β ∈ [V:x] , hence β(x) ∈ V . Thus V is a WE-quasi-prime submodule of X .

Proposition 3.6 : Let X be an R-module , and V is a WE-quasi-prime submodule of X . Then for every α , β ∈End (X), and x ∈ X.
[V : α o β(x)]= [(0) : α o β(x)] ∪ [V : α(x)] ∪ [V: β(x)].

Proof: Let a∈[V : α o β(x)] , implies that α o β(ax)∈ $V . If\ \alpha o\beta(ax)=0$, then
a∈ [(0) : α o β(x)]. So ,we assume that α o β(ax) ≠0 but V is a WE-quasi- prime then either α(ax) ∈ V or β(ax) ∈ V , hence either $a\in[V{:}\,\alpha(x)]$ or β ∈ [V : β(x)].
Consequently [V : α o β(x)]= [(0) : α o β(x)] ∪ [V : α(x)] ∪ [V : β(x)].

Proposition 3.7 : Let V is a WE-quasi-prime submodule of an R-module X , and W is an X-injective submodule of X .Then either W ⊆ Vor W⋂V is WE-quasi-prime submodule of W.

Proof: Assume that W⊈ V , then W⋂V is a Proper subset of W. Let 0≠ α o β(x) ∈ W⋂V, where x ∈ W and α, β ∈ End(W) , with β(x) ∉W⋂V , then β(x) ∉V . Since W is X-injective, then there exist , f_1 f_2 : X→W , such that $f_1$oi =α , $f_2$oi=β , where i: W→X is the inclusion map clearly f_1 f_2 ∈ End(X).
Now, 0≠ α o β(x) = ($f_1$oi) o ($f_2$oi) (x) ∈ V, implies that0 ≠ f_1 o f_2 (x) ∈ V . But V is a WE-quasi-prime f_2(x) = β(x) ∉V , implies that f_1(x)=α(x) ∈ V.
If f_1(x) ∈ V , then f_1(x) ∈ W⋂V, since f_1(x) ∈ W, hence α(x) ∈ W⋂V. Therefore W⋂V is a WE-quasi-prime submodule of W.

Proposition 3.8 : Let X be an R-module , and V , W submodules of X with W is a fully invariant submodule of X and W ⊆ V . If X/W is a WE-quasi-prime submodule of X/W , then V is a quasi-prime submodule of X .

Proof: Similarly as in proposition2.12.

Proposition 3.9: Let X be a duo R-module , and V be a proper submodule of X . Then V is a WE- quasi-prime submodule of X if and only if [V:x] =[V : α(x)] for each x ∈ X and α ∈ End(X) with α(x) ∉V.

Proof: (⇒) Let 0≠β ∈[V: α(x)] , α(x) ∉V then 0≠β(α(x))∈V, implies that 0≠β o α(x)∈V. But V is a WE-quasi-prime submodule of X, and α(x) ∉V , then β(x) ∈V . Thus β ∈ [V:x] , hence [V : α(x)] ⊆ [V:x] . Now, let 0≠β ∈[V : x] , implies that β(x) ∈V. But X is a duo module, then α(x) ∈< x>. It follows that β(α(x))⊆ β(x) ⊆ V ,implies that β ∈ [V: α(x)]. Hence [V : x] ⊆[V : α(x)] , thus , [V : x] = [V : α(x)].
(⇐) 0≠ α o β(x)∈V , x ∈ X, and α,β ∈ End (X) and β(x) ∉V . Since 0≠ α (β (x))∈V , implies that 0≠ α∈[V : β(x)] = [V : x] , implies that α∈[V : x] , hence α(x) ∈V . Thus V is a WE-quas-prime submodule of X .

Proposition 3.10: Let X be an R-module , and f ∈ End (X), and V be a fully invariant submodule of X . If V is a WE-quasi-prime submodule of X , then$f^{-1}(V)$is a WE-quasi-prime submodule of X.

Proof: Clearly f^{-1}(V) is a proper submodule of X , let 0≠ α o β (x) ∈ f^{-1}(V) , where x ∈ X . and α,β ∈ End (X) .
If follows that f(α o β (x)) ∈ V then ((foα)oβ) (x) ∈ V. Since V is a WE-quasi-prime submodule of X , then either (foα)(x) ∈ V or β (x) ∈ V, thus either α(x) ∈ f^{-1}(V) or f o β (x) ∈ f(V) ⊆ V because V is a fully invariant submodule of X , hence (f(β (x)) ∈ V, implies that β (x) ∈ f^{-1}(V) . So f^{-1}(V) is a WE-quasi-prime submodule of X .

Proposition 3.11: Let X be an R-module , and V be a WE-quasi-prime
submodule of X . Then [V:W] is weakly prime ideal of E=End(X) for every
submodule W of X , with W ⊈ V.

Proof: let 0≠ α o β(x)∈[V:W] , where α,β ∈ End (X), implies that 0≠ α o β(W) ⊆ V, hence by proposition 3.4 either α(W) ⊆ V or β(W) ⊆ V , hence either α∈[W:V] or β ∈[W:V] . Thus [W:V] is a weakly prime ideal of End (X).

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المقاسات الجزئية الابتدائية من النمط –WE والمقاسات الجزئية الأولية ظاهريا من النمط WE

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[2]قسم الرياضيات كلية التربية للعلوم الصرفة ، جامعة تكريت ، تكريت ،العراق

الملخص

في هذا البحث قدمنا ودرسنا مفهومين الاول هو صف جزئي من صف المقاسات الجزئية الابتدائية الضعيفة.
صف المقاييس الجزئية الظاهرية الضعيفة والذي يدعى المقياس الجزئي الابتدائي من النمط –WE والثاني هو صف جزئي من صف المقاسات الجزئية الأولية الظاهرية الضعيفة والذي يدعى المقياس الجزئي الاولي الظاهري من النمط –WE . اعطيت العديد من الصفات المقاسات الجزئية الابتدائية من النمط WE والمقاسات الجزئية الأولية الظاهرية من النمط – WE وبعض المكافئات لهذين النوعين من المقاسات الجزئية قدمت. بالإضافة الى ذلك سلوك هذه المقاسات الجزئية في بعض انواع المقاسات وجدت. من ناحيه اخرى الصورة والصورة العكسية لهذين المقياسين الجزئيين تحت تأثير التشاكل برهنت.