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Study of Some Kinds of Ridge Regression Estimators in Linear Regression Model

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ABSTRACT

In linear regression model, the biased estimation is one of the most commonly used methods to reduce the effect of the multicollinearity. In this paper, a simulation study is performed to compare the relative efficiency of some kinds of biased estimators as well as for twelve proposed estimated ridge parameter (k) which are given in the literature. We propose some new adjustments to estimate the ridge parameter. Finally, we consider a real data set in economics to illustrate the results based on the estimated mean squared error (MSE) criterion.

According to the results, all the proposed estimators of (k) are superior to ordinary least squared estimator (OLS), and the superiority among them based on minimum MSE matrix will change according to the sample under consideration.

1. Introduction

Let

$y = X\beta + \varepsilon$ ….(1.1)

be the multiple linear regression model, where y is an (n × 1) vector of responses, X is an (n × p) design matrix of the explanatory variables, p is the number of the explanatory variables , β is a (p × 1) vector of unknown parameters of interest, ε is an (n × 1) vector of residuals that follow the standard assumptions, namely, $E(\varepsilon) = 0$ and $E(\varepsilon'\varepsilon) = \sigma^2 I_n$. I_n is an identity matrix of order n.

The OLS of β is the best linear unbiased estimator (BLUE) which is given by

$\hat{\beta}_{OLS} = (X'X)^{-1}X'y$ …(1.2)

The most important assumption in multiple linear regression model, the explanatory variables must be considered as independent of each other. But, practically, there are probably linear dependencies between these variable values. Mainly, this problem could appear in econometric data and it's called multicollinearity. Multicollinearity influences the regression analysis extremely and it is one of the main problems. The existence of multicollinearity makes the estimates of the correlation coefficients large and very large sampling variances of the OLS estimated Lukman et al.[1]. To overcome this problem, there are various methods have been mentioned in literature and one of them is by using the biased estimators. The common biased estimation method is the ridge regression which was proposed by Hoerl and Kennard [2] and still the researchers working in this area like Kibria, and Banik [3]. They suggested using the ordinary ridge regression (ORR) as bellow:

$\hat{\beta}_R = (X'X + kI_p)^{-1}X'y$, ….(1.3)

where k is the ridge parameter and the value of $k > 0$ .The ORR estimator is biased to a certain value of k which is unknown and therefore it should be estimated from real data.

A number of ways for obtaining biased estimates of β with smaller MSE have been developed. By extending Hoerl and Kennard's model, Crouse et al. [4] defined the unbiased ridge regression (URR) estimator as follows:

$\hat{\beta}(kI, J) = (X'X + kI_p)^{-1}(X'y + kJ)$, …(1.4)

where J is a random vector with $J \sim N(\beta, (\sigma^2/k)I)$. Battah and Gore [5] proposed a modified unbiased ridge regression (MURR) estimator of β and still the researchers who work in this area like Lukman et al.[6]and Tarima et al. [7] which is denoted as below:

$\hat{\beta}_J(k) = [I - k(X'X + kI_p)^{-1}](X'X + kI_p)^{-1}(X'y + kJ)$ … (1.5) the ORR and URR estimators have been combined to obtain the MURR which was driven from ORR by using URR rather than OLS .

The two- parameter estimator (TPE) proposed by Ozkale and Kacıranlar [8] and still the researchers working in this area like Asar, and Genç [9]. which is denoted as follows:

$$\hat{\beta}(k, d) = (X'X + kI_p)^{-1}(X'y + kd\hat{\beta}_{OLS}) = F_{kd}\hat{\beta}_{OLS} \quad ....(1.6)$$

where $F_{kd} = (X'X + kI_p)^{-1}(X'X + kdI)$, $k > 0$ and d is shrinkage parametar such that $0 < d < 1$.

To simplify the considerations about the linear model, the canonical form is often used. Therefore, a symmetric matrix S = X′X has an eigenvalue–eigenvector decomposition of the form $S = T\Lambda T'$, where T is an orthogonal matrix and Λ is a real diagonal matrix. The diagonal elements of Λ are the eigenvalues of S and the column vectors of T are the eigenvectors of S. The orthogonal version of the regression model in (1-1) is

$$y = XTT'\beta + \varepsilon = Z\gamma + \varepsilon \quad ...(1.7)$$

where Z = XT, $\gamma = T'\beta$ and Z′Z = $\Lambda = dig(\lambda_1, \lambda_2, ..., \lambda_p)$.

The OLS estimator of γ is given by

$$\hat{\gamma}_{OLS} = (Z'Z)^{-1}Z'y = \Lambda^{-1}Z \ , \quad ...(1.8)$$

The goal of this paper is to compare the different biased estimators as well as with different estimated value of k using the MSE as a measure of goodness of fit.

The paper is organized as follows. In Section 2, we present the methodology of different estimators of k and propose some new estimators. A Monte Carlo simulation has been given in Section 3. The discussions of the results of the simulation are given in Section 4. Finally, in Section 5, a real data set as an application of this study is given.

2. Estimation of Ridge Parameter

Hoerl and Kennard [2] showed the properties of ORR in detail. They concluded that the total variance decreases and the squared bias increases as k increases. The variance function is monotonically decreasing and the squared bias function is monotonically increasing. That means, there is a chance that some k exists such that the MSE for ORR is less than MSE for the OLS.

It is well known that k is unknown and estimated from the sample of the study. For this reason, there are many articles proposed different ridge parameters in the literature using different techniques. Recently, many researchers studied this area and proposed different estimates of k. We review available methods in literatures to estimate the value of k as follows:

- Hoerl and Kennard [2] suggested k to be (denoted here by $\hat{k}_{HK}$)

$$\hat{k}_{HK} = \frac{\hat{\sigma}^2}{\hat{\gamma}^2_{\max OLS}}, \quad ....(2.1)$$

where $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}\hat{e}_i^2}{n-p}$ and $\hat{\gamma}_{\max OLS}$ is the maximum element of $\hat{\gamma}_{OLS}$

- Hoerl et al. [10] proposed k to be (denoted here by $\hat{k}_{HKB}$)

$$\hat{k}_{HKB} = \frac{p\hat{\sigma}^2}{\hat{\gamma}'_{OLS}\hat{\gamma}_{OLS}}, \quad .... (2.2)$$

- Lawless and Wang [11] suggested k to be (denoted here by $\hat{k}_{LW}$)

$$\hat{k}_{LW} = \frac{p\hat{\sigma}^2}{\hat{\gamma}'_{OLS}X'X\hat{\gamma}_{OLS}}, \quad ....(2.3)$$

- Hocking et al. [12] suggested k to be(denoted here by($\hat{k}_{HSL}$)

$$\hat{k}_{HSL} = \hat{\sigma}^2\frac{\sum_{i=1}^{P}(\lambda_i\hat{\gamma}_{i\,OLS})^2}{\left(\sum_{i=1}^{P}\lambda_i\hat{\gamma}^2_{i\,OLS}\right)^2}, \quad ...(2.4)$$

where $\hat{\gamma}_{i\,OLS}$ is the i^th^ element of $\hat{\gamma}_{OLS}$

- Nomura [13] suggested k to be (denoted by $\hat{k}_{HMO}$)

$$\hat{k}_{HMO} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\left[\hat{\gamma}^2_{i\,OLS}\Big/\left[1+\left(1+\lambda_i\left(\frac{\hat{\gamma}^2_{i\,OLS}}{\hat{\sigma}^2}\right)^{\frac{1}{2}}\right)\right]\right]}, \quad ....(2.5)$$

where λ_i is the i^th^ eigenvalues.

- Kibria [14] proposed the following estimators for k based on arithmetic mean (AM), geometric mean (GM), and median of $\hat{\sigma}^2/\hat{\gamma}_i^2$. These are defined as follows:

The estimator based on AM (denoted by $\hat{k}_{AM}$)

$$\hat{k}_{AM} = \frac{1}{p}\sum_{i=1}^{p}\frac{\hat{\sigma}^2}{\hat{\gamma}^2_{i\,OLS}} \quad ....(2.6)$$

The estimator based on GM (denoted by $\hat{k}_{GM}$)

$$\hat{k}_{GM} = \frac{\hat{\sigma}^2}{\left(\prod_{i=1}^{P}\hat{\gamma}^2_{i\,OLS}\right)^{\frac{1}{P}}} \quad .....(2.7)$$

The estimator based on median (denoted by $\hat{k}_{MED}$)

$$\hat{k}_{MED} = Median\left\{\frac{\hat{\sigma}^2}{\hat{\gamma}^2_{i\,OLS}}\right\}, \quad i=1,2,\dots,p \quad ...(2.8)$$

- Based on modification of $\hat{k}_{HK}$, Khalaf and Shukur [15] suggested k to be (denoted by $\hat{k}_{KS}$)

$$\hat{k}_{KS} = \frac{\lambda_{\max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{\max}\hat{\gamma}^2_{\max OLS}} \quad .....(2.9)$$

where $\lambda_{\max}$ is the maximum eigenvalue of the matrix **X'X**.

- Following Kibria [14] and Khalaf and Shukur [15], Alkhamisi et al. [16] proposed the following three estimators of k:

$$\hat{k}^{KS}_{arith}=\frac{1}{p}\sum_{i=1}^{p}\frac{\lambda_i\hat{\sigma}^2}{(n-p)\hat{\sigma}^2+\lambda_i\hat{\gamma}^2_{iOLS}} \quad \ldots(2.10)$$

$$\hat{k}^{KS}_{max}=\max\left(\frac{\lambda_i^2\hat{\sigma}^2}{(n-p)\hat{\sigma}^2+\lambda_i\hat{\gamma}^2_{iOLS}}\right) \quad i=1,\ldots,p \quad \ldots(2.11)$$

$$\hat{k}^{KS}_{md}=median\left(\frac{\lambda_i\hat{\sigma}^2}{(n-p)\hat{\sigma}^2+\lambda_i\hat{\gamma}^2_{iOLS}}\right) \quad i=1,\ldots,p \quad \ldots(2.12)$$

Now, we propose some new methods based as follows:

$$\hat{k}_{MU1}=\frac{\lambda_{med}\sum_{i=1}^{p}\hat{\gamma}^2_{iOLS}}{\lambda_{\max}} \quad \ldots(2.13)$$

$$\hat{k}_{MU2}=\left|\frac{p\hat{\sigma}^2}{\hat{\gamma}'_{OLS}\hat{\gamma}_{OLS}}-\frac{p\hat{\sigma}^2}{\hat{\gamma}'_{OLS}X'X\hat{\gamma}_{OLS}}\right| \quad \ldots(2.14)$$

$$\hat{k}_{MU3}=\min\left(\sqrt{\frac{\lambda_{\min}\sum_{i=1}^{p}\hat{\gamma}^2_{iOLS}}{\hat{\sigma}^2}}\right) \quad \ldots(2.15)$$

$$\hat{k}_{MU4}=\max\left(\sqrt{\frac{\lambda_{\min}\sum_{i=1}^{p}\hat{\gamma}^2_{iOLS}}{\hat{\sigma}^2}}\right) \quad \ldots(2.16)$$

$$\hat{k}_{MU5}=\max\left(\frac{\lambda_{\min}\sum_{i=1}^{p}\hat{\gamma}^2_{iOLS}}{\sqrt{\hat{\sigma}^2}}\right) \quad \ldots(2.17)$$

3. A simulation study

The aim of the current study is to perform a comparison of different biased estimators for variate estimates of ridge parameter which are given in (2.1-2.17) and identify some good estimators for practitioners. We conduct a simulation study using Matlab. This simulation has been designed depends on specific factors that are expected to influence the properties of estimators which be subjected to a statistical investigation Lukman et al.[17]. Since the degree of the collinearity among several explanatory variables (Xs) is very essential, Kibria [14] was followed to generate X's using the following equation:

$$X_{ij}=\left(1-\varphi^2\right)^{\frac{1}{2}}z_{ij}+\varphi z_{ip}, \quad i=1,2,\ldots,n, j=1,2,\ldots,p, \quad \ldots(3.1)$$

where the z_{ij} independent standard normal pseudo-random numbers and φ represents the correlation between any two X's. These various are standardized so that X'X is being in correlation forms. The response variable y is considered by

$$y_i=\beta_1X_{i1}+\beta_2X_{i2}+\ldots+\beta_pX_{ip}+e_i, \quad i=1,2,\ldots,n, \quad \ldots(3.2)$$

where the e_i is i.i.d. N(0, σ^2). Therefore, zero intercept for (3.2) will be assumed. Also the number of explanatory variables $p=5$, while the values of σ are chose as (1, 5, 10, 20). The correlation φ will choose as (0.75, 0.85, 0.90, 0.95) and sample size n=(50, 100, 150) .The coefficients β_1, β_2, …, β_p are selected as the eigenvectors corresponding to the largest eigenvalue of the matrix X'X subject to constraint $\beta'\beta=1$. Thus, for n, p, β, λ, φ, and σ, sets of Xs are created. Then the experiment was re-preformed 10000 times by creating new error terms. The estimated MSE for each estimator is calculated as follows:

$$mse(\beta^*)=\frac{1}{10000}\sum_{i=1}^{10000}(\beta^*-\beta)'(\beta^*-\beta), \cdots (3.3)$$

where β^* would be any of the estimators (OLS, ORR, MURR, or TPE).

4. The discussion of simulation results

In this section we present the results of our Monte Carlo experiment concerning the properties of the different methods used to choose the ridge parameter K, when multicollinearity among the columns of the design matrix of the explanatory variables exist. The simulation results are presented in Tables 1–12 and we will discuss the results by dividing the results in three parts:

4-1 The simulation results according to the different estimators

Table (4-1) shows an explanation of the preference of the estimators mentioned in this paper, where we can observe the following:

1- The MURR estimator is the best estimator that has the lowest MSE compared to the rest of the estimators in different sample sizes in all correlations and σ. This is what we note in Table (4-1) as well as Tables (1-12) attached in this paper.

2- In case (n=50, $\sigma=1$, φ =0.75, 0.90) and (n=100, 150, $\sigma=1$, φ =0.85) the ORR estimator is better than others which can give us an indicator for using it instead of MURR in case we need that.

Table 4-1: The simulation results according to the best estimators in each case

Table	n	σ	φ	Best estimator	Table	n	σ	φ	Best estimator
1-4	50	1	0.75	ORR	8-12	150	1	0.75	MURR
			0.85	MURR				0.85	ORR
			0.90	ORR				0.90	MURR
			0.95	MURR				0.95	MURR
		5	0.75	MURR			5	0.75	MURR
			0.85	MURR				0.85	MURR
			0.90	MURR				0.90	MURR
			0.95	MURR				0.95	MURR
		10	0.75	MURR			10	0.75	MURR
			0.85	MURR				0.85	MURR
			0.90	MURR				0.90	MURR
			0.95	MURR				0.95	MURR
		20	0.75	MURR			20	0.75	MURR
			0.85	MURR				0.85	MURR
			0.90	MURR				0.90	MURR
			0.95	MURR				0.95	MURR
4-8	100	1	0.75	MURR					
			0.85	ORR					
			0.90	MURR					
			0.95	MURR					
		5	0.75	MURR					
			0.85	MURR					
			0.90	MURR					
			0.95	MURR					
		10	0.75	MURR					
			0.85	MURR					
			0.90	MURR					
			0.95	MURR					
		20	0.75	MURR					
			0.85	MURR					
			0.90	MURR					
			0.95	MURR					

4-2 **The simulation results according to the different estimated ridge parameter**

In order to know the preference of the estimated ridge parameter that mentioned in this paper, Tables (4-2 to 4-5) show an explanation that, where we can observe the following:

1- By increasing the sample size, we observe others estimated of ridge parameter which gives lowest MSE and still MED, HKB, and LW give well performance as we observed in Table (4-2).

2- From Tables((4-3) to (4-5)) and Tables (1-12), the proposed estimated ridge parameter (MU1-MU5) are working well compared to other estimated ridge parameter, especially with MURR estimator and this is the case for all situations as well as it compared with OLS estimator.

3- From Tables((4-3) to (4-5)) in general we observe that all estimated ridge parameter working well with MURR estimator which is the best estimator according to this study, that means we can use any one of them to find the MURR estimator.

Table 4-2 The simulation results according to the different estimated ridge parameter

Table	n	σ	φ	Best estimator of k	Table	n	σ	φ	Best estimator of k
1-5	50	1	0.75	GM	10-15	150	1	0.75	HKB
			0.85	HKB				0.85	MU3
			0.90	HKB				0.90	MED
			0.95	GM				0.95	MU2
		5	0.75	MED			5	0.75	LW
			0.85	MED				0.85	MU5
			0.90	MU2				0.90	HMO
			0.95	GM				0.95	MU5
		10	0.75	MED			10	0.75	HSL
			0.85	AM				0.85	GM
			0.90	MU4				0.90	AM
			0.95	HMO				0.95	MU5
		20	0.75	MU4			20	0.75	MU2
			0.85	AM				0.85	GM
			0.90	AM				0.90	MED
			0.95	MU4				0.95	MU3
5-10	100	1	0.75	LW					
			0.85	HKB					
			0.90	HKB					
			0.95	GM					
		5	0.75	HK					
			0.85	LW					
			0.90	LW					
			0.95	MU2					
		10	0.75	MU3					
			0.85	LW					
			0.90	GM					
			0.95	HK					
		20	0.75	AM					
			0.85	MU4					
			0.90	AM					
			0.95	MED					

Table 4-3: The simulation results according to the best estimated ridge parameter when n=50

n	σ	φ	HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
50	1	0.75	MURR	MURR	MURR	MURR	TPE	--	ORR	ORR	MURR	MURR	MURR	MURR	MURR	MURR	TPE	--	TPE
		0.85	MURR	MURR	MURR	MURR	--	--	ORR	ORR	MURR	MURR	MURR	MURR	MURR	MURR	--	--	TPE
		0.90	MURR	ORR	ORR	MURR	--	--	TPE	TPE	MURR	MURR	MURR	MURR	MURR	ORR	--	--	TPE
		0.95	MURR	MURR	MURR	MURR	MURR	ORR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	TPE	ORR
	5		HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
		0.75	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	MURR
		0.85	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.90	MURR	MURR	MURR	MURR	MURR	ORR	ORR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	MURR
		0.95	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	ORR	MURR
	10		HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
		0.75	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.85	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.90	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.95	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
	20		HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
		0.75	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.85	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.90	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.95	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR

Table 4-4 The simulation results according to the best estimated ridge parameter when n=100

n	σ	φ	HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
100	1	0.75	MURR	ORR	MURR	MURR	--	--	--	--	MURR	MURR	MURR	MURR	TPE	MURR	--	--	--
		0.85	ORR	ORR	ORR	ORR	--	--	TPE	--	ORR	ORR	ORR	ORR	ORR	ORR	--	--	--
		0.90	MURR	MURR	MURR	MURR	--	--	TPE	--	MURR	MURR	MURR	MURR	MURR	MURR	--	--	--
		0.95	MURR	MURR	MURR	MURR	MURR	ORR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	TPE	--	ORR
	5		HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
		0.75	MURR	ORR	ORR	MURR	TPE	--	TPE	ORR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	TPE	MURR
		0.85	MURR	MURR	MURR	MURR	ORR	TPE	ORR	ORR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	ORR	MURR
		0.90	MURR	MURR	MURR	MURR	MURR	ORR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	ORR	MURR
		0.95	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
			HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
	10	0.75	MURR	MURR	MURR	MURR	ORR	--	ORR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	TPE	MURR
		0.85	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.90	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.95	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
			HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
	20	0.75	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.85	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.90	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.95	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR

Table 4-5 The simulation results according to the best estimated ridge parameter when n=150

n	σ	φ	HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
150	1	0.75	MURR	MURR	MURR	MURR	--	--	--	--	MURR	MURR	MURR	MURR	MURR	MURR	--	--	--
		0.85	MURR	MURR	MURR	MURR	--	--	--	--	ORR	ORR	ORR	ORR	ORR	ORR	ORR	--	--
		0.90	MURR	MURR	MURR	MURR	TPE	--	ORR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	--	--	--
		0.95	MURR	MURR	MURR	MURR	--	--	--	--	MURR	MURR	MURR	MURR	MURR	MURR	--	--	--
	5		HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
		0.75	MURR	MURR	MURR	MURR	ORR	TPE	ORR	ORR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	TPE	MURR
		0.85	MURR	MURR	MURR	MURR	MURR	TPE	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	ORR	MURR
		0.90	MURR	MURR	MURR	MURR	MURR	ORR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	MURR
		0.95	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
	10		HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
		0.75	MURR	MURR	MURR	MURR	MURR	--	MURR	TPE	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.85	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	ORR	ORR	MURR
		0.90	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.95	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
	20		HK	HKB	LW	HSL	HMO	AM	GM	MED	KS	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5
		0.75	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.85	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.90	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR
		0.95	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR	MURR

5. A numerical example

Real Life Application

In order to give more explanation for the study, we consider the data set in economics on total national research and development expenditures as a percent of gross national product originally due to Gruber [18] and later by Akdeniz and Erol [19], among others. This reflects the relationship between the dependent Y variable the percentage expended by the United States and the other four independent X1 , X2, X3, and X4 variables. The vector X1 reflects the amount that France spent, X2 that West Germany spent, X3 that Japan spent, and X4 that the former Soviet Union spent on.

The goal is to compare the traces of the estimated MSE matrices of (ORR), (MURR) and (TPE). The trace of the MSE matrix of the (ORR) is given by mse($\hat{\beta}_R$)=tr(MSE($\hat{\beta}_R,\beta$))= $\sum_{i=1}^{p}\frac{\lambda_i\sigma^2+k^2\beta_i^2}{(\lambda_i+k)^2}$, …(5.1)

the trace of the MSE matrix of the (MURR) is given by

mse($\hat{\beta}_J(k)$)=tr(MSE($\hat{\beta}_J(k),\beta$))=

$\sum_{i=1}^{p}\frac{\lambda_i\sigma^2+k^2(\lambda_i+k)\beta_i^2}{(\lambda_i+k)^3}$, …(5.2)

the trace of the MSE matrix of the (TPE) is given by

mse($\hat{\beta}(k,d)$)=tr(MSE($\hat{\beta}(k,d),\beta$))=

$\sum_{i=1}^{p}\frac{\lambda_i\sigma^2(\lambda_i+d)^2+((k+1-d)\lambda_i+k)^2\beta_i^2}{(\lambda_i+1)^2(\lambda_i+k)^2}$, …(5.3)

we are substituting β and σ^2 by their OLS estimates $\hat{\beta}$ and $\hat{\sigma}^2$ respectively. For the standardized data since there are ten observations and four parameters, we obtain $\hat{\sigma}^2 = 0.003932$. The four eigenvalues of X' X are 2.95743, 0.91272, 0.10984, and 0.02021. The factors will define a 4-dimensional space and the $X'X$ matrix will be as follows:

$$X'X=\begin{bmatrix}1.000 & 0.888 & .925 & 0.309\\ 0.888 & 1.000 & 0.962 & 0.157\\ 0.925 & 0.962 & 1.000 & 0.328\\ 0.309 & 0.157 & 0.328 & 1.000\end{bmatrix}$$

We can observe that the variables in $X'X$ matrix suffer for high correlations among them and this is the one advantage of standardizing the X matrix where it can be seen which variables are highly correlated. Another method for diagnosing multicollinearity in linear regression, is the Condition Index (C.I.) which is defined as follows:

$C.I.=\sqrt{\frac{\lambda_{max}}{\lambda_{min}}}$,

where λ_{max} and λ_{min} are the largest and the smallest eigenvalues of X' X , if C.I. ≤ 10 , then there is no multicollinearity among the explanatory variables, if 10 < C.I. < 30, then the multicollinearity is moderate, but if C.I .≥ 30, then it means that there is a severe multicollinearity that must be corrected. So in this example , $10 < C.I.=\sqrt{\frac{2.95743}{0.02021}} = 12.1 < 30$, which indicates that there is a moderate multicollinearity and may be corrected.

Table (5-1): The scaler mean squares error for different estimators and different estimated ridge parameter

	HK	HKB	LW	HSL	HMO	AM	GM	MED	KS
OLS	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361
ORR	0.1166	0.1140	0.1213	0.1321	0.1591	0.2963	0.1356	0.1137	0.1310
MURR	0.0880	0.0876	0.0910	0.1006	0.1503	0.2950	0.1227	0.0879	0.0996
TPE	0.1565	0.1618	0.1518	0.1464	0.2270	0.3338	0.2045	0.1631	0.1467
	KS arith	KS max	KS md	MU1	MU2	MU3	MU4	MU5	
OLS	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	
ORR	0.1180	0.1771	0.1352	0.1603	0.1468	0.3018	0.3230	0.1556	
MURR	0.0990	0.1703	0.1037	0.1516	0.1162	0.3006	0.3221	0.1462	
TPE	0.1814	0.2420	0.1454	0.2280	0.1427	0.3385	0.3572	0.2239	

From Table (5-1), we can observe that, the minimum mse for the ORR estimator will be got if k is estimated by HKB. Also the minimum mse for the MURR estimator will be given by estimating k by HKB while the minimum mse for the TPE estimator will be given by estimating k by MU2. The performance of the estimated k that given in this study is showing that under moderate degree of multicollinearity, the most of them give minimum mse if they used in the MURR estimator except (AM, MU3 and MU4) where the OLS estimator is better than of them. Therefore, not all proposed ridge parameter can be used to get minimum mse when the degree of multicollinearity is moderate.

Finally, we can say that, this study gives us a broad view on the behaviour of the estimators and when they can be used to give a good performance compared to the other suggested estimators.

Appendix

Table1: estimated MSEs when n=50 σ=1

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	0.3849	0.3756	0.3741	0.3801	0.3480	0.3437	0.3395	0.3458	0.3712	0.3655	0.3654	0.3683	0.6797	0.5886	0.5244	0.6320
HKB	0.3849	0.3619	0.3619	0.3724	0.3480	0.3360	0.3307	0.3414	0.3712	0.3568	0.3609	0.3632	0.6797	0.5198	0.4482	0.5919
LW	0.3849	0.3723	0.3707	0.3784	0.3480	0.3431	0.3385	0.3455	0.3712	0.3645	0.3645	0.3677	0.6797	0.5568	0.4848	0.6141
HSL	0.3849	0.3763	0.3749	0.3805	0.3480	0.3438	0.3396	0.3458	0.3712	0.3657	0.3655	0.3684	0.6797	0.5720	0.5027	0.6227
HMO	0.3849	0.3792	0.4251	0.3706	0.3480	0.3658	0.4110	0.3497	0.3712	0.3968	0.4476	0.3747	0.6797	0.4299	0.4139	0.5230
AM	0.3849	0.4730	0.5682	0.4061	0.3480	0.3723	0.4228	0.3524	0.3712	0.4827	0.5749	0.4107	0.6797	0.4227	0.4294	0.5096
GM	0.3849	0.3548	0.3720	0.3646	0.3480	0.3365	0.3504	0.3388	0.3712	0.3667	0.3962	0.3633	0.6797	0.4626	0.4134	0.5528
MED	0.3849	0.3561	0.3601	0.3684	0.3480	0.3380	0.3545	0.3392	0.3712	0.3724	0.4067	0.3653	0.6797	0.4873	0.4248	0.5707
KS	0.3849	0.3776	0.3763	0.3811	0.3480	0.3441	0.3403	0.3460	0.3712	0.3665	0.3662	0.3688	0.6797	0.6035	0.5455	0.6402
KS arith	0.3849	0.3803	0.3794	0.3826	0.3480	0.3461	0.3439	0.3470	0.3712	0.3689	0.3687	0.3700	0.6797	0.6454	0.6138	0.6623
KS max	0.3849	0.3776	0.3763	0.3811	0.3480	0.3441	0.3403	0.3460	0.3712	0.3665	0.3662	0.3688	0.6797	0.5861	0.5209	0.6306
KS md	0.3849	0.3807	0.3798	0.3827	0.3480	0.3465	0.3447	0.3472	0.3712	0.3695	0.3693	0.3703	0.6797	0.6630	0.6465	0.6713
MU1	0.3849	0.3691	0.3675	0.3766	0.3480	0.3441	0.3402	0.3460	0.3712	0.3666	0.3663	0.3688	0.6797	0.6087	0.5532	0.6429
MU2	0.3849	0.3688	0.3673	0.3764	0.3480	0.3381	0.3321	0.3427	0.3712	0.3596	0.3615	0.3650	0.6797	0.6000	0.5404	0.6383
MU3	0.3849	0.3866	0.4380	0.3731	0.3480	0.4460	0.5444	0.3844	0.3712	0.3990	0.4512	0.3756	0.6797	0.5160	0.6140	0.5256
MU4	0.3849	0.5885	0.7098	0.4523	0.3480	0.5404	0.6775	0.4254	0.3712	0.5626	0.6772	0.4445	0.6797	0.5373	0.6449	0.5337
MU5	0.3849	0.3985	0.4579	0.3774	0.3480	0.3562	0.3931	0.3458	0.3712	0.3859	0.4298	0.3704	0.6797	0.4227	0.4297	0.5095

Table2: estimated MSEs when n=50 σ=5

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	0.9738	0.9203	**0.8576**	0.9461	1.0148	0.9700	**0.9465**	0.9918	1.3535	1.1834	**1.0656**	1.2641	1.3518	0.9261	**0.7519**	1.1177
HKB	0.9738	0.8649	**0.7769**	0.9143	1.0148	0.8925	**0.8547**	0.9473	1.3535	1.0183	**0.9005**	1.1633	1.3518	0.7066	**0.6132**	0.9591
LW	0.9738	0.8571	**0.7694**	0.9094	1.0148	0.8481	**0.8192**	0.9154	1.3535	0.9016	**0.8653**	1.0632	1.3518	0.6250	**0.6033**	0.8654
HSL	0.9738	0.9303	**0.8767**	0.9515	1.0148	0.9018	**0.8639**	0.9531	1.3535	1.0957	**0.9634**	1.2131	1.3518	0.6351	**0.6010**	0.8821
HMO	0.9738	0.8081	**0.7742**	0.8651	1.0148	0.8342	**0.8122**	0.9016	1.3535	0.9086	**0.8638**	1.0719	1.3518	0.6278	**0.6023**	0.8706
AM	0.9738	0.8094	**0.7865**	0.8614	1.0148	0.8347	**0.8206**	0.8865	1.3535	**0.9960**	0.9973	1.0435	1.3518	0.6178	**0.6101**	0.8467
GM	0.9738	0.8284	**0.7529**	0.8893	1.0148	0.8328	**0.8118**	0.8997	1.3535	**0.8903**	0.8909	1.0263	1.3518	0.6422	**0.6006**	0.8919
MED	0.9738	0.8256	**0.7526**	0.8871	1.0148	0.8327	**0.8117**	0.8996	1.3535	0.9059	**0.8643**	1.0687	1.3518	0.6356	**0.6009**	0.8828
KS	0.9738	0.9496	**0.9169**	0.9615	1.0148	0.9917	**0.9782**	1.0031	1.3535	1.2389	**1.1466**	1.2943	1.3518	0.9824	**0.8068**	1.1520
KS arith	0.9738	0.9590	**0.9383**	0.9664	1.0148	1.0034	**0.9964**	1.0091	1.3535	1.2746	**1.2055**	1.3132	1.3518	1.0572	**0.8914**	1.1956
KS max	0.9738	0.9390	**0.8942**	0.9560	1.0148	0.9758	**0.9547**	0.9948	1.3535	1.1324	**1.0023**	1.2350	1.3518	0.7369	**0.6246**	0.9849
KS md	0.9738	0.9645	**0.9510**	0.9691	1.0148	1.0112	**1.0088**	1.0130	1.3535	1.3267	**1.3004**	1.3400	1.3518	1.3114	**1.2767**	1.3315
MU1	0.9738	0.9200	**0.8571**	0.9460	1.0148	0.9839	**0.9665**	0.9990	1.3535	1.2140	**1.1087**	1.2809	1.3518	1.0980	**0.9431**	1.2186
MU2	0.9738	0.9514	**0.9208**	0.9624	1.0148	0.8796	**0.8429**	0.9389	1.3535	0.9236	**0.8636**	1.0879	1.3518	0.6536	**0.6012**	0.9060
MU3	0.9738	0.8101	**0.7627**	0.8708	1.0148	0.8363	**0.8224**	0.8863	1.3535	1.1560	**1.0302**	1.2486	1.3518	**0.6944**	0.7302	0.8380
MU4	0.9738	**0.8278**	0.8298	0.8576	1.0148	0.8735	**0.8591**	0.8941	1.3535	**0.8957**	0.9012	1.0222	1.3518	**0.6623**	0.6902	0.8279
MU5	0.9738	0.8225	**0.7527**	0.8844	1.0148	0.8294	**0.8127**	0.8914	1.3535	0.9077	**0.8640**	1.0709	1.3518	0.6249	**0.6033**	0.8654

Table3: estimated MSEs when n=50 σ=10

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	1.1559	1.0719	**1.0158**	1.1117	1.2612	1.1348	**1.0660**	1.1910	1.4981	1.3011	**1.0854**	1.3947	1.4086	0.9914	**0.8873**	1.1659
HKB	1.1559	0.9888	**0.9292**	1.0605	1.2612	1.0642	**1.0155**	1.1377	1.4981	1.1461	**0.9141**	1.3032	1.4086	0.8898	**0.8436**	1.0776
LW	1.1559	0.9537	**0.9131**	1.0322	1.2612	1.0329	**0.9689**	1.1042	1.4981	0.9700	**0.8414**	1.1727	1.4086	0.8677	**0.8567**	1.0093
HSL	1.1559	1.0822	**1.0303**	1.1174	1.2612	1.1212	**1.0550**	1.1821	1.4981	1.2906	**1.0699**	1.3888	1.4086	0.8568	**0.8438**	1.0205
HMO	1.1559	0.9402	**0.9158**	1.0151	1.2612	1.0410	**0.9847**	1.1132	1.4981	0.9789	**0.8431**	1.1809	1.4086	0.8590	**0.8412**	1.0318
AM	1.1559	0.9387	**0.9240**	1.0063	1.2612	1.0140	**0.9580**	1.0869	1.4981	0.9254	**0.8305**	1.1216	1.4086	0.8686	**0.8576**	1.0092
GM	1.1559	0.9478	**0.9128**	1.0260	1.2612	1.0353	**0.9737**	1.1069	1.4981	1.0327	**0.8563**	1.2252	1.4086	0.8579	**0.8417**	1.0283
MED	1.1559	0.9495	**0.9127**	1.0279	1.2612	1.0378	**0.9785**	1.1096	1.4981	1.1496	**0.9166**	1.3054	1.4086	0.8944	**0.8447**	1.0826
KS	1.1559	1.1188	**1.0880**	1.1370	1.2612	1.1908	**1.1297**	1.2242	1.4981	1.3641	**1.1916**	1.4290	1.4086	1.0698	**0.9453**	1.2198
KS arith	1.1559	1.1332	**1.1133**	1.1444	1.2612	1.2239	**1.1839**	1.2421	1.4981	1.4067	**1.2765**	1.4514	1.4086	1.1753	**1.0532**	1.2840
KS max	1.1559	1.1015	**1.0596**	1.1279	1.2612	1.1628	**1.0938**	1.2082	1.4981	1.2334	**0.9958**	1.3562	1.4086	0.9402	**0.8604**	1.1257
KS md	1.1559	1.1415	**1.1284**	1.1486	1.2612	1.2447	**1.2247**	1.2529	1.4981	1.4673	**1.4172**	1.4826	1.4086	1.3782	**1.3533**	1.3933
MU1	1.1559	1.0968	**1.0522**	1.1253	1.2612	1.2208	**1.1783**	1.2405	1.4981	1.2980	**1.0808**	1.3930	1.4086	1.2645	**1.1702**	1.3338
MU2	1.1559	0.9956	**0.9342**	1.0653	1.2612	1.0374	**0.9778**	1.1092	1.4981	1.0153	**0.8513**	1.2116	1.4086	0.8644	**0.8537**	1.0099
MU3	1.1559	0.9482	**0.9127**	1.0265	1.2612	1.0836	**1.0297**	1.1546	1.4981	0.9178	**0.8262**	1.1071	1.4086	0.8651	**0.8544**	1.0097
MU4	1.1559	0.9409	**0.9303**	1.0030	1.2612	1.0405	**0.9838**	1.1127	1.4981	0.9156	**0.8201**	1.0866	1.4086	0.8611	**0.8505**	1.0114
MU5	1.1559	0.9579	**0.9140**	1.0362	1.2612	1.0979	**1.0388**	1.1657	1.4981	0.9694	**0.8413**	1.1721	1.4086	0.9200	**0.8524**	1.1079

Table 4: estimated MSEs when n=50 σ=20

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	1.2166	1.1419	**1.0906**	1.1775	1.3050	1.1664	**1.0972**	1.2291	1.4136	1.2456	**1.1721**	1.3203	1.5691	1.2542	**1.1181**	1.3996
HKB	1.2166	1.0459	**0.9907**	1.1181	1.3050	1.1028	**1.0467**	1.1862	1.4136	1.1703	**1.1157**	1.2667	1.5691	1.0288	**0.9323**	1.2493
LW	1.2166	1.0089	**0.9709**	1.0874	1.3050	1.0547	**1.0177**	1.1439	1.4136	1.1140	**1.0636**	1.2160	1.5691	0.9034	**0.8651**	1.1008
HSL	1.2166	1.1476	**1.0988**	1.1806	1.3050	1.2061	**1.1423**	1.2526	1.4136	1.1934	**1.1315**	1.2845	1.5691	1.0003	**0.9160**	1.2258
HMO	1.2166	0.9965	**0.9671**	1.0729	1.3050	1.0542	**1.0174**	1.1434	1.4136	1.1234	**1.0752**	1.2248	1.5691	0.9291	**0.8806**	1.1538
AM	1.2166	0.9997	**0.9991**	1.0469	1.3050	1.0169	**0.9888**	1.1027	1.4136	1.1074	**1.0549**	1.2099	1.5691	0.9055	**0.8636**	1.0924
GM	1.2166	0.9932	**0.9798**	1.0493	1.3050	1.0535	**1.0168**	1.1426	1.4136	1.1378	**1.0902**	1.2384	1.5691	0.9232	**0.8779**	1.1458
MED	1.2166	0.9928	**0.9788**	1.0496	1.3050	1.0865	**1.0370**	1.1733	1.4136	1.1699	**1.1154**	1.2664	1.5691	0.9774	**0.9038**	1.2055
KS	1.2166	1.1795	**1.1487**	1.1976	1.3050	1.2393	**1.1887**	1.2709	1.4136	1.3165	**1.2496**	1.3624	1.5691	1.3221	**1.1951**	1.4387
KS arith	1.2166	1.1950	**1.1758**	1.2057	1.3050	1.2754	**1.2484**	1.2900	1.4136	1.3674	**1.3260**	1.3899	1.5691	1.3763	**1.2641**	1.4687
KS max	1.2166	1.1573	**1.1131**	1.1859	1.3050	1.2155	**1.1545**	1.2578	1.4136	1.2777	**1.2035**	1.3400	1.5691	1.0943	**0.9758**	1.2977
KS md	1.2166	1.2053	**1.1949**	1.2109	1.3050	1.2950	**1.2851**	1.3000	1.4136	1.4026	**1.3911**	1.4081	1.5691	1.5391	**1.5152**	1.5540
MU1	1.2166	1.1802	**1.1499**	1.1980	1.3050	1.2753	**1.2483**	1.2899	1.4136	1.3821	**1.3519**	1.3976	1.5691	1.3844	**1.2750**	1.4731
MU2	1.2166	1.0399	**0.9867**	1.1137	1.3050	1.0728	**1.0291**	1.1615	1.4136	1.1283	**1.0807**	1.2295	1.5691	0.9039	**0.8664**	1.1066
MU3	1.2166	1.1760	**1.1427**	1.1958	1.3050	1.1139	**1.0538**	1.1944	1.4136	1.1224	**1.0740**	1.2239	1.5691	0.9035	**0.8646**	1.0988
MU4	1.2166	0.9932	**0.9666**	1.0677	1.3050	1.0534	**1.0168**	1.1426	1.4136	1.1223	**1.0739**	1.2238	1.5691	0.9068	**0.8635**	1.0906
MU5	1.2166	1.0493	**0.9930**	1.1206	1.3050	1.1622	**1.0931**	1.2266	1.4136	1.2494	**1.1755**	1.3226	1.5691	0.9440	**0.8876**	1.1717

Table5: estimated MSEs when n=100 σ=1

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	0.2884	0.2883	**0.2882**	0.2884	0.3230	**0.3213**	0.3213	0.3221	0.3497	0.3479	**0.3459**	0.3488	0.4632	0.4409	**0.4221**	0.4518
HKB	0.2884	**0.2883**	0.2885	0.2883	0.3230	**0.3173**	0.3185	0.3200	0.3497	0.3443	**0.3404**	0.3467	0.4632	0.4128	**0.3829**	0.4363
LW	0.2884	0.2883	**0.2882**	0.2884	0.3230	**0.3207**	0.3208	0.3218	0.3497	0.3476	**0.3454**	0.3486	0.4632	0.4395	**0.4199**	0.4511
HSL	0.2884	0.2883	**0.2882**	0.2884	0.3230	**0.3213**	0.3214	0.3221	0.3497	0.3479	**0.3459**	0.3488	0.4632	0.4414	**0.4230**	0.4520
HMO	**0.2884**	0.3312	0.3793	0.3066	**0.3230**	0.3363	0.3744	0.3243	**0.3497**	0.3761	0.4202	0.3580	0.4632	0.3647	**0.3638**	0.4008
AM	**0.2884**	0.3170	0.3530	0.3004	**0.3230**	0.5167	0.6385	0.4005	**0.3497**	0.3666	0.4020	0.3540	0.4632	**0.3749**	0.4083	0.3967
GM	**0.2884**	0.2948	0.3064	0.2909	0.3230	0.3274	0.3579	**0.3209**	0.3497	0.3463	0.3564	**0.3460**	0.4632	0.3719	**0.3572**	0.4088
MED	**0.2884**	0.3035	0.3259	0.2945	**0.3230**	0.3580	0.4114	0.3331	**0.3497**	0.3779	0.4236	0.3587	0.4632	0.3814	**0.3583**	0.4163
KS	0.2884	0.2883	**0.2882**	0.2884	0.3230	**0.3215**	0.3216	0.3222	0.3497	0.3482	**0.3465**	0.3489	0.4632	0.4440	**0.4273**	0.4534
KS arith	0.2884	0.2883	**0.2883**	0.2884	0.3230	**0.3220**	0.3220	0.3225	0.3497	0.3491	**0.3483**	0.3494	0.4632	0.4568	**0.4505**	0.4600
KS max	0.2884	0.2883	**0.2882**	0.2884	0.3230	**0.3215**	0.3216	0.3222	0.3497	0.3482	**0.3465**	0.3489	0.4632	0.4440	**0.4273**	0.4534
KS md	0.2884	0.2883	**0.2882**	0.2884	0.3230	**0.3221**	0.3221	0.3225	0.3497	0.3492	**0.3487**	0.3495	0.4632	0.4600	**0.4568**	0.4616
MU1	0.2884	0.2883	0.2886	**0.2883**	0.3230	**0.3178**	0.3187	0.3203	0.3497	0.3477	**0.3456**	0.3487	0.4632	0.4444	**0.4281**	0.4536
MU2	0.2884	0.2883	**0.2882**	0.2883	0.3230	**0.3188**	0.3193	0.3208	0.3497	0.3452	**0.3414**	0.3473	0.4632	0.4270	**0.4009**	0.4443
MU3	**0.2884**	0.4734	0.6027	0.3685	**0.3230**	0.3559	0.4080	0.3323	**0.3497**	0.5032	0.6280	0.4142	0.4632	0.4910	0.6198	**0.4428**
MU4	**0.2884**	0.6718	0.8263	0.4507	**0.3230**	0.6648	0.7993	0.4617	**0.3497**	0.5795	0.7288	0.4473	**0.4632**	0.5490	0.7057	0.4680
MU5	**0.2884**	0.4223	0.5289	0.3465	**0.3230**	0.4291	0.5205	0.3633	**0.3497**	0.3764	0.4208	0.3581	0.4632	**0.3716**	0.4004	0.3961

Table6: estimated MSEs when n=100 σ=5

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	0.8592	0.8317	**0.8280**	0.8428	0.8607	0.8046	**0.7642**	0.8314	0.9929	0.8740	**0.8070**	0.9295	1.0191	0.8400	**0.8063**	0.9158
HKB	0.8592	**0.8346**	0.8467	0.8408	0.8607	0.7360	**0.7002**	0.7891	0.9929	0.7899	**0.7323**	0.8761	1.0191	0.8015	**0.7815**	0.8836
LW	0.8592	**0.8367**	0.8519	0.8412	0.8607	0.7262	**0.6984**	0.7816	0.9929	0.7512	**0.7205**	0.8440	1.0191	0.7877	**0.7788**	0.8589
HSL	0.8592	0.8313	**0.8292**	0.8422	0.8607	0.7501	**0.7073**	0.7990	0.9929	0.7772	**0.7262**	0.8665	1.0191	0.7880	**0.7767**	0.8650
HMO	0.8592	0.8664	0.9013	**0.8514**	0.8607	**0.7115**	0.7286	0.7596	0.9929	0.7390	**0.7256**	0.8291	1.0191	0.7873	**0.7778**	0.8605
AM	**0.8592**	0.9185	0.9573	0.8734	0.8607	0.8336	0.9129	**0.7974**	0.9929	**0.7640**	0.7932	0.8163	1.0191	0.7873	**0.7772**	0.8622
GM	0.8592	0.8610	0.8938	**0.8493**	0.8607	**0.7172**	0.7473	0.7581	0.9929	0.7410	**0.7237**	0.8321	1.0191	0.7905	**0.7770**	0.8700
MED	0.8592	**0.8392**	0.8575	0.8418	0.8607	**0.7293**	0.7738	0.7595	0.9929	0.7443	**0.7219**	0.8364	1.0191	0.7881	**0.7794**	0.8582
KS	0.8592	0.8519	**0.8453**	0.8555	0.8607	0.8298	**0.8031**	0.8449	0.9929	0.9225	**0.8722**	0.9564	1.0191	0.9103	**0.8718**	0.9608
KS arith	0.8592	0.8566	**0.8539**	0.8579	0.8607	0.8463	**0.8324**	0.8534	0.9929	0.9608	**0.9342**	0.9766	1.0191	0.9733	**0.9505**	0.9956
KS max	0.8592	0.8514	**0.8445**	0.8552	0.8607	0.8138	**0.7775**	0.8364	0.9929	0.8868	**0.8228**	0.9368	1.0191	0.8821	**0.8427**	0.9439
KS md	0.8592	0.8580	**0.8567**	0.8586	0.8607	0.8555	**0.8503**	0.8581	0.9929	0.9853	**0.9784**	0.9891	1.0191	1.0134	**1.0101**	1.0163
MU2	0.8592	0.8426	**0.8320**	0.8503	0.8607	0.8036	**0.7627**	0.8308	0.9929	0.7977	**0.7369**	0.8816	1.0191	0.7889	**0.7767**	0.8672
MU3	0.8592	**0.8317**	0.8370	0.8408	0.8607	**0.7105**	0.7131	0.7639	0.9929	**0.7425**	0.7574	0.8141	1.0191	0.8444	**0.8322**	0.8707
MU4	0.8592	0.8464	0.8710	**0.8440**	0.8607	**0.7673**	0.8349	0.7711	0.9929	**0.7718**	0.8042	0.8183	1.0191	0.8010	**0.7924**	0.8562
MU5	0.8592	0.8398	**0.8292**	0.8487	0.8607	0.7111	**0.7099**	0.7655	0.9929	0.7487	**0.7208**	0.8414	1.0191	0.8313	**0.7999**	0.9093

Table7: estimated MSEs when n=100 σ=10

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	0.7725	0.7156	**0.6918**	0.7416	1.1059	1.0206	**0.9795**	1.0584	1.1204	1.0415	**1.0106**	1.0760	1.0430	0.9450	**0.9329**	0.9820
HKB	0.7725	0.7005	**0.6787**	0.7318	1.1059	0.9978	**0.9638**	1.0424	1.1204	1.0074	**0.9913**	1.0492	1.0430	0.9375	**0.9345**	0.9703
LW	0.7725	0.7029	**0.6804**	0.7335	1.1059	0.9759	**0.9560**	1.0203	1.1204	0.9964	**0.9831**	1.0319	1.0430	0.9569	**0.9568**	0.9727
HSL	0.7725	0.7250	**0.7023**	0.7472	1.1059	1.0509	**1.0119**	1.0767	1.1204	1.0503	**1.0179**	1.0817	1.0430	0.9425	**0.9419**	0.9681
HMO	0.7725	**0.6893**	0.6993	0.7137	1.1059	0.9766	**0.9560**	1.0215	1.1204	0.9981	**0.9860**	1.0370	1.0430	0.9413	**0.9405**	0.9680
AM	**0.7725**	0.9012	0.9373	0.7912	1.1059	0.9813	**0.9646**	1.0114	1.1204	0.9998	**0.9985**	1.0283	1.0430	0.9612	**0.9611**	0.9745
GM	0.7725	**0.6851**	0.6855	0.7152	1.1059	0.9801	**0.9568**	1.0263	1.1204	0.9964	**0.9826**	1.0308	1.0430	0.9436	**0.9431**	0.9683
MED	0.7725	0.7045	**0.6816**	0.7346	1.1059	0.9976	**0.9637**	1.0423	1.1204	0.9973	**0.9851**	1.0353	1.0430	0.9393	**0.9381**	0.9682
KS	0.7725	0.7538	**0.7418**	0.7629	1.1059	1.0811	**1.0577**	1.0932	1.1204	1.0858	**1.0593**	1.1024	1.0430	0.9858	**0.9585**	1.0119
KS arith	0.7725	0.7657	**0.7609**	0.7691	1.1059	1.0970	**1.0874**	1.1014	1.1204	1.1070	**1.0941**	1.1136	1.0430	1.0203	**1.0029**	1.0313
KS max	0.7725	0.7526	**0.7401**	0.7623	1.1059	1.0768	**1.0503**	1.0909	1.1204	1.0768	**1.0468**	1.0974	1.0430	0.9773	**0.9507**	1.0066
KS md	0.7725	0.7690	**0.7665**	0.7707	1.1059	1.1028	**1.0993**	1.1043	1.1204	1.1170	**1.1135**	1.1187	1.0430	1.0404	**1.0381**	1.0417
MU1	0.7725	0.7531	**0.7409**	0.7626	1.1059	1.0942	**1.0819**	1.1000	1.1204	1.1108	**1.1013**	1.1156	1.0430	1.0370	**1.0316**	1.0400
MU2	0.7725	0.7622	**0.7551**	0.7673	1.1059	0.9813	**0.9571**	1.0276	1.1204	0.9966	**0.9838**	1.0331	1.0430	0.9543	**0.9542**	0.9717
MU3	0.7725	0.6932	**0.6750**	0.7264	1.1059	0.9910	**0.9606**	1.0369	1.1204	1.0603	**1.0275**	1.0878	1.0430	0.9384	**0.9368**	0.9686
MU4	0.7725	0.7276	0.7621	**0.7226**	1.1059	0.9749	**0.9560**	1.0179	1.1204	0.9974	**0.9852**	1.0355	1.0430	0.9387	**0.9371**	0.9684
MU5	0.7725	0.6965	**0.6763**	0.7290	1.1059	1.0338	**0.9920**	1.0667	1.1204	1.0366	**1.0071**	1.0727	1.0430	0.9787	**0.9519**	1.0075

Table 8: estimated MSEs when n=100 σ=20

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	1.1688	1.1194	**1.0871**	1.1427	1.6168	1.5161	**1.3951**	1.5655	1.2001	1.0818	**1.0274**	1.1382	1.2033	1.0200	**0.9389**	1.1048
HKB	1.1688	1.0703	**1.0333**	1.1124	1.6168	1.3166	**1.0840**	1.4564	1.2001	0.9294	**0.8754**	1.0434	1.2033	0.8719	**0.8186**	1.0046
LW	1.1688	1.0434	**1.0105**	1.0914	1.6168	1.0238	**0.8904**	1.2577	1.2001	0.8530	**0.8291**	0.9668	1.2033	0.8151	**0.8028**	0.9336
HSL	1.1688	1.1285	**1.1000**	1.1478	1.6168	1.4413	**1.2593**	1.5259	1.2001	0.8869	**0.8474**	1.0091	1.2033	0.8211	**0.8005**	0.9511
HMO	1.1688	1.0367	**1.0044**	1.0853	1.6168	1.0735	**0.9053**	1.2981	1.2001	0.8622	**0.8338**	0.9828	1.2033	0.8220	**0.8006**	0.9526
AM	1.1688	1.0112	**0.9859**	1.0604	1.6168	0.9309	**0.8558**	1.1314	1.2001	0.8524	**0.8288**	0.9577	1.2033	0.8631	**0.8488**	0.9347
GM	1.1688	1.0332	**1.0010**	1.0820	1.6168	1.0015	**0.8851**	1.2375	1.2001	0.8561	**0.8307**	0.9736	1.2033	0.8153	**0.8014**	0.9377
MED	1.1688	1.0510	**1.0171**	1.0979	1.6168	1.0392	**0.8945**	1.2708	1.2001	0.8524	**0.8288**	0.9648	1.2033	0.8188	**0.8004**	0.9469
KS	1.1688	1.1541	**1.1414**	1.1614	1.6168	1.5400	**1.4436**	1.5778	1.2001	1.1189	**1.0764**	1.1582	1.2033	1.0710	**1.0001**	1.1339
KS arith	1.1688	1.1624	**1.1566**	1.1656	1.6168	1.5463	**1.4568**	1.5811	1.2001	1.1495	**1.1205**	1.1743	1.2033	1.1215	**1.0701**	1.1612
KS max	1.1688	1.1501	**1.1345**	1.1593	1.6168	1.3809	**1.1667**	1.4929	1.2001	1.0375	**0.9749**	1.1130	1.2033	0.9679	**0.8867**	1.0731
KS md	1.1688	1.1656	**1.1626**	1.1672	1.6168	1.5993	**1.5751**	1.6081	1.2001	1.1912	**1.1855**	1.1956	1.2033	1.1915	**1.1825**	1.1974
MU1	1.1688	1.1564	**1.1455**	1.1625	1.6168	1.3935	**1.1848**	1.4999	1.2001	1.1339	**1.0977**	1.1662	1.2033	1.1126	**1.0571**	1.1566
MU2	1.1688	1.0609	**1.0253**	1.1056	1.6168	1.0609	**0.9010**	1.2883	1.2001	0.8576	**0.8315**	0.9762	1.2033	0.8159	**0.8009**	0.9401
MU3	1.1688	1.0764	**1.0387**	1.1165	1.6168	0.9341	**0.8637**	1.1513	1.2001	0.8674	**0.8372**	0.9523	1.2033	0.8162	**0.8051**	0.9299
MU4	1.1688	1.0278	**0.9955**	1.0766	1.6168	0.9400	**0.8548**	1.1190	1.2001	0.8772	**0.8439**	0.9539	1.2033	0.8408	**0.8286**	0.9277
MU5	1.1688	1.0753	**1.0378**	1.1158	1.6168	0.9366	**0.8657**	1.1571	1.2001	0.8546	**0.8300**	0.9708	1.2033	0.8230	**0.8007**	0.9541

Table 10: estimated MSEs when n=150 σ=5

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	0.8010	0.7789	**0.7617**	0.7893	0.9006	0.8659	**0.8460**	0.8827	0.9968	0.9321	**0.8937**	0.9633	1.3652	1.1826	**1.0720**	1.2696
HKB	0.8010	0.7617	**0.7497**	0.7782	0.9006	0.8181	**0.7920**	0.8545	0.9968	0.8299	**0.7805**	0.9030	1.3652	1.0188	**0.8954**	1.1727
LW	0.8010	0.7620	**0.7496**	0.7784	0.9006	0.8048	**0.7826**	0.8450	0.9968	0.7824	**0.7525**	0.8668	1.3652	0.8251	**0.7898**	1.0153
HSL	0.8010	0.7745	**0.7564**	0.7868	0.9006	0.8334	**0.8063**	0.8642	0.9968	0.8132	**0.7684**	0.8914	1.3652	0.8762	**0.8056**	1.0682
HMO	0.8010	**0.7718**	0.8004	0.7749	0.9006	0.7908	**0.7844**	0.8284	0.9968	0.7687	**0.7500**	0.8522	1.3652	0.8781	**0.8064**	1.0698
AM	0.8010	0.7949	0.8406	**0.7823**	0.9006	**0.8039**	0.8085	0.8267	0.9968	**0.8146**	0.8229	0.8441	1.3652	0.8630	**0.8004**	1.0561
GM	0.8010	**0.7596**	0.7670	0.7732	0.9006	0.7911	**0.7811**	0.8309	0.9968	0.7628	**0.7557**	0.8394	1.3652	0.9288	**0.8317**	1.1107
MED	0.8010	**0.7586**	0.7598	0.7739	0.9006	0.7983	**0.7797**	0.8396	0.9968	0.7628	**0.7557**	0.8393	1.3652	0.9014	**0.8170**	1.0893
KS	0.8010	0.7949	**0.7884**	0.7979	0.9006	0.8861	**0.8761**	0.8932	0.9968	0.9573	**0.9310**	0.9766	1.3652	1.2368	**1.1480**	1.2990
KS arith	0.8010	0.7986	**0.7959**	0.7998	0.9006	0.8945	**0.8901**	0.8975	0.9968	0.9761	**0.9613**	0.9863	1.3652	1.2877	**1.2278**	1.3257
KS max	0.8010	0.7940	**0.7866**	0.7974	0.9006	0.8801	**0.8667**	0.8901	0.9968	0.9247	**0.8834**	0.9593	1.3652	1.1063	**0.9798**	1.2262
KS md	0.8010	0.7998	**0.7984**	0.8004	0.9006	0.8984	**0.8967**	0.8995	0.9968	0.9918	**0.9880**	0.9943	1.3652	1.3574	**1.3505**	1.3613
MU1	0.8010	0.7945	**0.7876**	0.7977	0.9006	0.8857	**0.8756**	0.8930	0.9968	0.9513	**0.9219**	0.9735	1.3652	1.2399	**1.1527**	1.3006
MU2	0.8010	0.7994	**0.7975**	0.8002	0.9006	0.8486	**0.8233**	0.8731	0.9968	0.8207	**0.7736**	0.8967	1.3652	0.8390	**0.7928**	1.0319
MU3	0.8010	**0.7661**	0.7876	0.7735	0.9006	**0.8036**	0.8080	0.8266	0.9968	0.7650	**0.7624**	0.8355	1.3652	0.8642	**0.8572**	0.9732
MU4	0.8010	0.8141	0.8676	**0.7895**	0.9006	**0.8335**	0.8443	0.8351	0.9968	**0.8218**	0.8306	0.8468	1.3652	0.8372	**0.8306**	0.9651
MU5	0.8010	0.7590	**0.7540**	0.7753	0.9006	0.7935	**0.7792**	0.8347	0.9968	0.7629	**0.7562**	0.8389	1.3652	0.8161	**0.7889**	1.0024

Table 11: estimated MSEs when n=150 σ=10

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	0.9528	0.9355	**0.9293**	0.9423	1.0269	0.9748	**0.9367**	1.0000	1.1496	1.0754	**1.0435**	1.1085	1.4702	1.3290	**1.2539**	1.3974
HKB	0.9528	0.9404	**0.9312**	0.9419	1.0269	0.8984	**0.8454**	0.9558	1.1496	1.0389	**1.0204**	1.0817	1.4702	1.1019	**0.9934**	1.2674
LW	0.9528	0.9514	**0.9397**	0.9460	1.0269	0.8503	**0.8176**	0.9210	1.1496	1.0170	**1.0047**	1.0569	1.4702	0.8556	**0.8193**	1.0680
HSL	0.9528	0.9363	**0.9288**	0.9411	1.0269	0.9037	**0.8501**	0.9592	1.1496	1.0754	**1.0435**	1.1085	1.4702	0.9347	**0.8643**	1.1489
HMO	0.9528	0.9546	**0.9425**	0.9473	1.0269	0.8389	**0.8172**	0.9098	1.1496	1.0243	**1.0129**	1.0665	1.4702	0.9288	**0.8606**	1.1439
AM	**0.9528**	0.9838	0.9758	0.9604	1.0269	0.8330	**0.8211**	0.9015	1.1496	1.0058	**0.9948**	1.0467	1.4702	0.8702	**0.8272**	1.0867
GM	0.9528	0.9587	**0.9464**	0.9490	1.0269	0.8477	**0.8171**	0.9188	1.1496	1.0211	**1.0100**	1.0623	1.4702	0.9331	**0.8633**	1.1476
MED	0.9528	0.9640	0.9518	**0.9514**	1.0269	0.8345	**0.8194**	0.9040	1.1496	1.0280	**1.0153**	1.0709	1.4702	0.8904	**0.8381**	1.1085
KS	0.9528	0.9493	**0.9475**	0.9510	1.0269	1.0004	**0.9783**	1.0134	1.1496	1.1202	**1.0972**	1.1344	1.4702	1.3616	**1.2999**	1.4146
KS arith	0.9528	0.9516	**0.9510**	0.9522	1.0269	1.0145	**1.0034**	1.0206	1.1496	1.1389	**1.1289**	1.1442	1.4702	1.3834	**1.3321**	1.4260
KS max	0.9528	0.9491	**0.9471**	0.9509	1.0269	0.9835	**0.9501**	1.0046	1.1496	1.1125	**1.0858**	1.1302	1.4702	1.1799	**1.0714**	1.3146
MU1	0.9528	0.9521	**0.9517**	0.9525	1.0269	0.9964	**0.9714**	1.0114	1.1496	1.1393	**1.1297**	1.1444	1.4702	1.3000	**1.2148**	1.3819
MU2	0.9528	0.9417	**0.9322**	0.9424	1.0269	0.8804	**0.8317**	0.9440	1.1496	1.0182	**1.0064**	1.0584	1.4702	0.8659	**0.8248**	1.0815
MU3	0.9528	0.9362	**0.9302**	0.9430	1.0269	**0.8573**	0.8725	0.8936	1.1496	1.0381	**1.0201**	1.0810	1.4702	0.8770	**0.8229**	1.0171
MU4	0.9528	0.9403	**0.9312**	0.9419	1.0269	**0.8674**	0.8846	0.8963	1.1496	1.0186	**1.0069**	1.0590	1.4702	0.8713	**0.8192**	1.0157
MU5	0.9528	0.9427	**0.9383**	0.9474	1.0269	0.8338	**0.8201**	0.9030	1.1496	1.0597	**1.0316**	1.0980	1.4702	0.8393	**0.8095**	1.0371

Table 12: estimated MSEs when n=150 σ=20

φ	0.75				0.85				0.90				0.95			
	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE	OLS	ORR	MURR	TPE
HK	1.0183	0.9882	**0.9702**	1.0022	1.0772	1.0385	**1.0159**	1.0561	1.1236	1.0705	**1.0417**	1.0952	1.1797	1.0281	**0.9799**	1.0934
HKB	1.0183	0.9644	**0.9491**	0.9861	1.0772	1.0146	**0.9963**	1.0394	1.1236	1.0209	**0.9994**	1.0616	1.1797	0.9933	**0.9611**	1.0669
LW	1.0183	0.9592	**0.9497**	0.9787	1.0772	1.0038	**0.9863**	1.0280	1.1236	1.0016	**0.9812**	1.0386	1.1797	0.9636	**0.9555**	1.0232
HSL	1.0183	0.9862	**0.9678**	1.0010	1.0772	1.0474	**1.0264**	1.0614	1.1236	1.0747	**1.0468**	1.0977	1.1797	0.9956	**0.9620**	1.0688
HMO	1.0183	0.9599	**0.9514**	0.9777	1.0772	1.0046	**0.9874**	1.0290	1.1236	1.0052	**0.9879**	1.0450	1.1797	0.9681	**0.9540**	1.0405
AM	1.0183	0.9739	**0.9677**	0.9793	1.0772	1.0000	**0.9999**	1.0197	1.1236	1.0000	**1.0000**	1.0309	1.1797	0.9688	**0.9593**	1.0220
GM	1.0183	0.9612	**0.9533**	0.9772	1.0772	1.0006	**0.9842**	1.0215	1.1236	1.0000	**0.9931**	1.0311	1.1797	0.9687	**0.9541**	1.0414
MED	1.0183	0.9645	**0.9573**	0.9772	1.0772	1.0070	**0.9903**	1.0321	1.1236	1.0001	**0.9754**	1.0330	1.1797	0.9987	**0.9634**	1.0714
KS	1.0183	1.0114	**1.0055**	1.0148	1.0772	1.0679	**1.0586**	1.0725	1.1236	1.1016	**1.0848**	1.1123	1.1797	1.1058	**1.0579**	1.1409
KS arith	1.0183	1.0157	**1.0133**	1.0170	1.0772	1.0740	**1.0705**	1.0756	1.1236	1.1151	**1.1079**	1.1193	1.1797	1.1520	**1.1284**	1.1656
KS max	1.0183	1.0102	**1.0033**	1.0142	1.0772	1.0664	**1.0558**	1.0717	1.1236	1.0932	**1.0719**	1.1079	1.1797	1.0861	**1.0333**	1.1297
KS md	1.0183	1.0172	**1.0161**	1.0177	1.0772	1.0760	**1.0748**	1.0766	1.1236	1.1218	**1.1202**	1.1227	1.1797	1.1764	**1.1733**	1.1781
MU1	1.0183	1.0144	**1.0110**	1.0163	1.0772	1.0739	**1.0702**	1.0755	1.1236	1.1161	**1.1097**	1.1198	1.1797	1.1577	**1.1384**	1.1685
MU2	1.0183	0.9612	**0.9481**	0.9829	1.0772	1.0061	**0.9893**	1.0310	1.1236	1.0024	**0.9833**	1.0403	1.1797	0.9630	**0.9550**	1.0239
MU3	1.0183	0.9618	**0.9482**	0.9836	1.0772	1.0686	**1.0599**	1.0728	1.1236	1.1215	**1.1196**	1.1226	1.1797	0.9625	**0.9538**	1.0292
MU4	1.0183	0.9623	**0.9547**	0.9771	1.0772	1.0040	**0.9866**	1.0283	1.1236	1.0013	**0.9801**	1.0378	1.1797	0.9628	**0.9549**	1.0242
MU5	1.0183	0.9735	**0.9550**	0.9930	1.0772	1.0369	**1.0143**	1.0551	1.1236	1.0280	**1.0038**	1.0673	1.1797	1.0172	**0.9729**	1.0857

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دراسة بعض أنواع تقديرات انحدار الحرف في نموذج الانحدار الخطي

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الملخص

يعد التقدير المتحيز أحد أكثر الأساليب المستخدمة شيوعًا لتقليل تأثير مشكلة تعدد العلاقات الخطية على تقدير المعلمات في نماذج الانحدار الخطي المتعددة. في هذا البحث، تم إجراء دراسة محاكاة لدراسة الكفاءة النسبية لبعض أنواع المقدرات المتحيزة بالإضافة إلى اثنا عشر معلمة مقدرة مقترحة لمعامل الحرف (k) مذكورة في البحوث. تم اقتراح بعض الانواع الجديدة لتقدير معلمة مقدرة لمعامل الحرف (k). أخيرًا ، تم استخدام مجموعة بيانات حقيقية لتوضيح النتائج استنادًا إلى معيار لمتوسط الخطأ المقدر. وفقًا للنتائج، فإن جميع المقدرات المقترحة لـ (k) أفضل من المُقدِّر بطريقة المربعات الصغرى (LSE)، ولكن لا يوجد ضمان للمُقدِّر "الأمثل"، وسيتوقف الاختيار الأفضل للمُقدِّر على الشروط الدراسة.