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On Micro $-\dot{b}$ $-$ open sets and Micro $-\dot{b}$ $-$ continuity

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ABSTRACT

The purpose of this research is generalize the concepts of micro $-\dot{b}-$ openset, so we study the relation this new concept open set micro-pre-open set, micro-semi-open set, micro$-\alpha-$ open set, and micro $-\beta-$openset and micro $-\dot{b}-$open continuity and micro $-\dot{b}-$ irresolute continuity in micro topological space and study the relationship among them, many theorems were proved as characteristic of these type of continuity and some examples were introduced.

1 Introduction

In 2013 Thivagar [1] introduced nano topology; based on the concept of lower approximation, upper approximation and boundary region. Nano topology have maximum five nano open sets and minimum three nano open sets including $U, \emptyset$ and " on nano – open sets " introduced in 2015 by Revalthy .A and Ganambal llango [4]

In 1996 Andrijevic [6] introduced a study about " b - open sets "and "on nano $b-$ open sets in nano topological spaces" introduced in 2016 by M.parimala, C.indirani and S.jafari [7]. Andrijevic. D.[5] in 1987 introduced research " on the topology generated by pre – open set " and " on semi pre – open sets in nano topological spaces " introduced by Mary .D .A and Arokia Rani. A .[8] in 2014 .

Suppose we want add some more open sets ,For that in 2019 S. Chandrasekar [3] introduced " on Micro topological spaces " which is extension concept of nano topological space such that we have some more open sets and every nano topological space is Micro topological space.

Levine [9] introduced semi – opensets and semi continuity in topological spaces in 1963. Mashhour [10] introduced a study about on pre – continuous mappings and weak pre continuous mappings in 1982 . Power .P.L and Rajak . K [12] introduced " Fine irresolute mappings " in year 2012 and " A new of contra contiuity in Bi – supra topological space " introduced by Taha. H. J. and others [11] in 2018 .

In this paper we define micro $-\dot{b}-$ open set, and study the relationships with micro – pre – open set micro – semi – open set, micro – α – openset and micro - open set .

At last micro $-\dot{b}$ $-$ continuity and micro $-\dot{b}$ $-$ irresolute were studied and investigated some characterization.

The important of this paper to generalize some class of open sets and contiuoutiy in order to extension this subject to a new spaces .

2 Preliminary

Now recall some definition and examples which is useful in our work.

Definition2.1. [1]

Let U be an non- empty finite set of objects named the universe and R be an equivalence relationship on U called the indiscernibility relation. Elements of the same category of equality are said to be indiscernible from each other. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$, Then :

1. The lower approximation of X relative to R is a set of all objects, which can be classified as X relative to R and is defined by $L_R(x) = \bigcup_{x \in U} \{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class specified by x

2. The upper approximation of X relative to R is a set of all objects, which can be possibly classified as X

relative to R and is defined by $U_R(X) = \bigcup_{x\in U}\{R(x): R(x)\cap X \neq \emptyset\}$.
3. The boundary area of X relative to R is the set ofall objects, which can be classified neither as X nur as not X relative to R and is defined by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 [1]

Let U be the universe, R be an equivalence relation on U and $T_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $T_R(X)$ satisfies the following axioms:

1. U and $\emptyset \in T_R(X)$.
2. The union of elements of any subset of $T_R(X)$ is in $T_R(X)$.
3. The intersection of elements of any finite subset of $T_R(X)$ is in $T_R(X)$.

That is $T_R(X)$ forms a topology on U is named the nano topology on U with respect to X. We called $(U, T_R(X))$ the nano topological space. The element's of $T_R(X)$ is said to be nano open sets.

Definition 2.3 [3]

The pair $(U, T_R(X))$ is nano topological space here $\mu_R(X) = \{N \cup (N' \cap \mu)\}: N, N' \in T_R(X)$ and called it micro topology of $T_R(X)$ by μ where $\mu \notin T_R(X)$.

Definition 2.4 [3]

The micro topology $\mu_R(X)$ satisfy the following axioms :

1. $U, \emptyset \in \mu_R(X)$
2. The union of elements of any sub collection of $\mu_R(X)$ is in $\mu_R(X)$
3. The intersection of elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is named the micro topology on U relative to X. The triple $(U, T_R(X), \mu_R(X))$ is said to be micro topological spaces and the elements of $\mu_R(X)$ are named micro open sets and the complement of micro open sets is said an micro closed set .

Defintion 2.5.[3]

let $(U, T_R(X), \mu_R(X))$ be a micro topological space and $A \subset U$. then A is called to be micro$-$ pre $-$ open set if $A \subseteq Mic - int(Mic - cl(A))$ and micro – pre – closed set if $Mic - cl(Mic - int(A)) \subseteq A$. A complement of micro – pre – open set of U is said to be micro – pre – closed set .

Definition 2.6 [3]

Let $(U, T_R(X), \mu_R(X))$ be an micro topological space and $A \subset U$, then A is said to be micro - semi $-$ open set if $A \subseteq Mic - cl(Mic - int(A))$ and micro _ semi_closed set if $Mic - int(Mic - cl(A)) \subseteq A$. A complement of micro – semi – open set of a space U is said micro – semi – closed set .

3 Micro $- \dot{b}$ $-$ open sets

In this section we defined and study micro $- \dot{b} -$ open sets' some of their properties are analogous for open sets .and the relation it by micro$-$pre $-$open, micro -semi-open and micro$-\alpha-$open sets, and micro - β- open sets .

Definition 3.1:

suppose $(U, T_R(X), \mu_R(X))$ be a micro topological space and $A \subseteq U$, Then A is said to be micro$- \dot{b}$ $-$open if $A \subseteq Mic - int(Mic - cl(A)) \cup Mic - cl(Mic - int(A))$, We denoted by Mic$- \dot{b} -$ open .

Definition 3.2 :

Let $(U, T_R(X), \mu_R(X))$ be a micro topological space and $A \subseteq U$, then A be called $micro - \dot{b} - closed$ if its complement is $Mic - \dot{b} - open$ set and denoted by($Mic - \dot{b} - closed$).

Example 3.3:

Let $U = \{p, q, r, s, t\}$
$U/R = \{\{p\}, \{q, r, s\}, \{t\}\}$
And let $X = \{p, q\} \subseteq U$

Then $T_R(X) = \{U, \emptyset, \{p\}, \{t\}, \{p, t\}, \{p, q, r, s\}, \{q, r, s\}\}$
If $\mu = \{t\}$
$Micro - O = \mu_R(X) = \{U, \emptyset, \{p\}, \{t\}, \{p, t\}, \{p, q, r, s\}, \{q, r, s\}, \{q, r, s, t\}\}$
micro closed sets are
$U, \emptyset, \{p\}, \{p, t\}, \{t\}, \{q, r, s\}, \{p, q, r, s\}$ and $\{q, r, s, t\}$
$A = \{q, r, s\}$ then A is $Mic - \dot{b} - open$

Theorem 3.4:

Every Mic $-$ open set is $Mic - \dot{b}$ – open set .

Proof:

Let A be $Mic - open$, then $A = Mic - int(A) \subseteq Mic - cl(Mic - int(A))$
But; $A \subseteq Mic - cl(A)$, since $A = Mic - int(A)$
Then $A \subseteq Mic\ cl(Mic - int(A))$
Hence $A \subseteq Mic - cl(Mic - int(A)) \cup Mic - int(Mic - cl(A))$ ◻

The opposite of the upside theorem is not right, can be see in the next example .

Example 3.4.

Let $U = \{1, 2, 3, 4, 5\}$ and $U/R = \{\{1\}, \{2, 3, 4\}, \{5\}\}$
supoos $X = \{2, 3\} \subseteq U$
hence $T_R(X) = \{U, \emptyset, \{2, 3, 4\}\}$
Then $\mu = \{1\}$ then $Mic - O = \mu_R(X) = \{U, \emptyset, \{1\}, \{1, 2, 3, 4\}, \{2, 3, 4\}\}$
Clearly $A = \{1, 2, 3, 4\}$ is $Mic - \dot{b} - open$ but not Mic $-$ open .

Theorem 3.5:

Every Mic $-$ pre $-$open set is Mic $-\dot{b}$ $-$open set .

Proof:

Let A be a mic – pre – open set in $(U, T_R(X), \mu_R(X))$, then $A \subseteq Mic - Int(Mic - cl(A))$
And since $Mic - Int(Mic - cl(A)) \subseteq Mic - Int(Mic - cl(A)) \cup Mic - cl(Mic - Int(A))$, then $A \subseteq Mic - Int(Mic - Cl(A)) \cup$

$Mic - cl\left(Mic - Int(A)\right)$, Hence A is $Mic - \dot{b} -$ open set in $(U, \mathrm{T}_R(X), \mu_R(X))$ □

The opposite of the upside theory is not right as shown in the following example.

Example 3.6:

let $U = \{1,2,3,4,5\}$, $U/R = \{\{1\},\{2,3,4\},\{5\}\}$ and let $X = \{2,3\}$then $\mathrm{T}_R(X) = \{U, \emptyset, \{2,3,4\}\}$, then $\mu = \{1\}$, then $Mic - O = \mu_R(X) = \{U, \emptyset, \{1\}, \{1,2,3,4\}, \{2,3,4\}\}$,

Here A $\{1,2,3,5\}$ is $Mic - b -$ open set but not mic – pre – open set .

Theorem 3.7:

Every Mic – semi – open set is Mic – $\dot{b}$ – open set.

Proof:

suppose A be a mic – semi – open set in $(U, \mathrm{T}_R(X), \mu_R(X))$.

Then $A \subseteq Mic - cl\,(Mic - Int\,(A))$.

And since $Mic - cl\left(Mic - Int(A)\right) \subseteq Mic - cl\left(Mic - Int\,(A)\right) \cup Mic - Int\left(Mic - cl\,(A)\right)$

Then $A \subseteq Mic - cl\left(Mic - Int\,(A)\right) \cup Mic - Int\left(Mic - cl\,(A)\right)$,

$Hence$ A is $Mic - \dot{b} -$ open set in $(U, \mathrm{T}_R(X), \mu_R(X))$ □

The opposite of the upside theorem need not to be right as shown by the following example.

Example3.8:

Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\} \subseteq U$

Then$\mathrm{T}_R(X) = \{U, \emptyset, \{a\}, \{a,b,d\}, \{b,d\}\}$, then $\mu = \{d\}$,

then $Mic - O = \mu_R(X) = \{U, \emptyset, \{a\}, \{d\}, \{a,d\}, \{a,b,d\}\}$,

Here $A = \{a,c,d\}$ is $Mic - b -$ open set but it is not Mic – semi – open.

Definition 3.9:

Suppose $(U, \mathrm{T}_R(X), \mu_R(X))$ be a micro topological space and $A \subset U$, then A is said tobe micro – α – openset briefly ($Mic - \alpha - open$) if $A \subseteq Mic - int(Mic - cl\,(Mic - int(A)$.

Definition 3.10 :

Let $(U, \mathrm{T}_R(X), \mu_R(X))$ be an micro topological space a subset A of U be called micro - α - closed set if its complement is Micro – α – open set

Briefly ($Mic - \alpha - closed$ set) .

Theorem 3.11:

Every Mic – α – open set is Mic – $\dot{b}$ – open set.

Proof:

Let A be a Mic – α – openset in $(U, \mathrm{T}_R(X), \mu_R(X))$, then

$A \subseteq Mic - int\,(Mic - cl\,(Mic - int(A))$, and since $Mic - int\,(Mic - cl\,(Mic - int\,(A)) \subseteq Mic - cl(Mic - int\,(A)) \cup Mic - cl\,(Mic - int\,(A))$

Then $A \subseteq Mic - cl\left(Mic - int\,(A)\right) \cup Mic - int\left(Mic - cl(A)\right)$,

Hence A is Mic $- \dot{b} -$ openset in $(U, \mathrm{T}_R(X), \mu_R(X))$ □

The opposite of the upside theorem needed not to be right as shown by the following example.

Example 3.12:

In the Example(3.6), a set A is mic $- \dot{b} -$ open set but not Mic – α – open set.

Defintion 3.13:

Let $(U, \mathrm{T}_R(X), \mu_R(X))$ be a micro topological space and $A \subset U$, then A is said to be micro $- \beta -$ open set briefly ($Mic - \beta - open\ set$) if $A \subseteq Mic - cl(Mic - int\,(Mic - cl(A))$.

Defintion 3.14 :

Let $(U, \mathrm{T}_R(X), \mu_R(X))$ be an micro topological space and $A \subset U$, then A is said to be micro $- \beta -$ $closed\ set$ briefly ($Mic - \dot{b} - closed\ set$) if its complement is Mic $- \beta -$ open set .

Theorem 3.15 :

Every Mic$-\dot{b}-$ open set is Mic$-\beta-$ open set.

Proof:

$Let\ A\ be\ Mic - \dot{b} - open\ set\ in\ (U, \mathrm{T}_R(X), \mu_R(X))$.

$Then\ A \subseteq Mic - cl(Mic - int(A)) \cup Mic - int(Mic - cl(A))$

$Mic - int(Mic - cl(A) \subseteq Mic - cl(Mic - int\left(Mic - cl\,(A)\right)$ ….. (1)

Since $A \subseteq Mic - cl\,(A)$ then $Mic - int\,(A) \subseteq Mic - int\left(Mic - cl(A)\right)$, thus $Mic - cl\left(Mic - int\,(A)\right) \subseteq Mic - cl\,(\,Mic - int\left(Mic - cl\,(A)\right)$ ….. (2)

From (1) and (2) we get A is Mic $- \beta -$ open set □

The opposite of the upside theorem is not right shown by the following example

Example 3.16:

In Example (3.6), a set $A = \{3,4,5\}$ is Mic $-\beta-$ open set but not Mic$-$ b – open set.

Remark 3.17 :

Intersection of two Mic$-\dot{b}$ $-$open sets needed not to be Mic $- \dot{b} -$ open set .

Example3.18:

Let $U = \{1,2,3,4\}$ with U / R = $\{\{1\},\{2,3\},\{4\}\}$, and $X = \{1,3\} \subseteq U$

$\mathrm{T}_R(X) = \{U, \emptyset, \{1,2,3\}, \{2,3\}, \{1\}\}$ $then$ $\mu = \{3\}$

$Micro - O = \mu_R(X) = \{U, \emptyset, \{1\}, \{2,3\}, \{1,2,3\}, \{3\}, \{1,3\}\}$

Then $\{1,4\}$ and $\{3,4\}$ are $Mic - \dot{b} - open\ sets$, but $\{4\}$$is\ not$ Mic $- \dot{b} -$ open set.

Definition3.19:

Let $(U, \mathrm{T}_R(X), \mu_R(X))$ be a micro topological space an element $x \in A$ is called Mic $-\dot{b}$ $-$interior point of

A if there exist a micro $-\dot{b}$ – open set H such that $x \in H \subseteq U$

Definition 3.20:
Let $(U, \mathrm{T}_R(X), \mu_R(X))$ be micro topological space and $A \subseteq U$, then the union of all $Mic - \dot{b} - open$ sets in U which contained in A be named a micro - $\dot{b}$ – interior of A and denoted by $(Mic - \dot{b} - interior)$.

Definition 3.21:
Let $(U, \mathrm{T}_R(X), \mu_R(X))$ be micro topological space and $A \subseteq U$, then the intersection of all $Mic - \dot{b} - closde$ sets including A be called aMicro - $\dot{b} - closure$ of A and denoted by $Mic - Cl_{\dot{b}}(A)$.

Remark 3.22:
It is clear that $Mic - int_b(A)$ is Mic$- b^{\cdot}$ $-$ open set and $Mic - cl_b(A)$ is Mic $-\dot{b}$ – closed set.

Theorem 3.23:
1. $A \subseteq Mic - cl_b(A)$ and $A = Mic - cl_b(A)$ if and only if A is $Mic - \dot{b} -$ closed set.
2. $Mic - int_b(A) \subseteq A$ and $A = Mic - int_b(A)$ if and only if A is $Mic - \dot{b} - open\ set$.

Theorem 3.24:
1. Arbitrary union of Mic$-\dot{b}$ $-$open sets is Mic $-\dot{b}$ $-$open set.
2. Arbitrary intersection of Mic$-\dot{b}$ $-$closed sets is Mic $-\dot{b}$ $-$closed set.

Proof:
1. Let $\{A_\alpha \mid \alpha \in I\}, I = 1, 2, \dots, n$
The family of Mic $-$
$\dot{b}$ – open sets in U. By definition (3.20)
This implies that $A_\alpha \subseteq [(Mic - int(Mic - cl(A_\alpha)) \cup (Mic - Cl(Mic - int(A_\alpha))] \ \forall \alpha \in I$
Then $\cup A_\alpha \subseteq \cup [(Mic - int(Mic - cl(A_\alpha)) \cup (Mic - cl(Mic - int(A_\alpha))] \ \forall \alpha \in I$.
$Since \bigcup(Mic - int(Mic - cl(A_\alpha)) \subseteq Mic - int(\bigcup Mic - cl(A_\alpha))\ and$
$Mic - int(\bigcup Mic - cl(A_\alpha)) \subseteq (Mic - int(Mic - cl(\bigcup A_\alpha)$ and by the same way we get
$\bigcup(Mic - cl(Mic - int(A_\alpha)) \subseteq (Mic - cl(Mic - int(\bigcup A_\alpha))$
$This\ implies\ that:$
$\bigcup A_\alpha \subseteq Mic - int(Mic - cl(\bigcup A_\alpha))\bigcup Mic - int(Mic - cl(\bigcup A_\alpha))$
Hence $\bigcup A_\alpha$ $is\ Mic - \dot{b} -$ open
□
2. Let $\{B_\alpha \mid \alpha \in I\}$, I $= 0, 1, 2, \dots, n$ be a family of Mic $-\dot{b}$ $-$ closed sets in U.
$suppose\ A_\alpha = B_\alpha^c,\ \forall \alpha \in I, then\ \{A_\alpha \mid \alpha \in I\}$ is a family of Mic $-\dot{b}$ $-$ sets, then by (1)
$\bigcup A_\alpha = \cup B_\alpha^c$ is Mic $-\dot{b}$ $-$ open,
Hence $(\bigcap B_\alpha)$ is Mic $-\dot{b}$ $-$ closed set □

Theorem 3.25:
Let $A \subset U\ (U, \mathrm{T}_R(X), \mu_R(X))$ then Mic $-\dot{b}$ $-$ $int(A)$ is equal to the union of all Mic $-\dot{b}$ – open sets contained in A.

Proof:
We need to prove that Mic $-\dot{b}$ $-$ $int(A) = \bigcup\{B \mid B \subset A, B\ is\ Mic - \dot{b} - open\ set\}$.
Let $x \in Mic - \dot{b} - int\ (A), then\ there\ exist\ Mic - \dot{b} - open\ set\ B_0\ in\ U$ such that $x \in B_0 \subseteq Mic - \dot{b} - int\ (A)$ by definition (3.20)
Conversely, suppose $x \in U\{B \mid B \subset A, B\ is\ Mic - \dot{b} - open\ set\}$ then there exist a set B_0 is Mic $-\dot{b}$ $-$open set, such that $x \in B_0 \subseteq A$.
Hence $x \in Mic - \dot{b} - int(A)$
And $\bigcup\{B \mid B \subset A, B\ is\ Mic - \dot{b} - open\ set\} \subset A$
So $Mic - \dot{b} - int(A) = \bigcup\{B \mid B \subset A, B\ is\ Mic - \dot{b} - open\ set\}$ □

Theorem 3.26:
1. $Mic - int_{\dot{b}}(A \bigcup B) \supset Mic - int_{\dot{b}}(A) \bigcup Mic - int_{\dot{b}}(B)$.
2. $Mic - int_{\dot{b}}(A \bigcap B) = Mic - int_{\dot{b}}(A) \bigcap Mic - int_{\dot{b}}(B)$.

Proof:
1. since $A \subset A \cup B$, then $Mic - int_{\dot{b}}(A) \cup Mic - int_{\dot{b}}(A \cup B)$ …… (1)
and $B \subset A \cup B$, then $Mic - int_{\dot{b}}(B) \subset Mic - int_{\dot{b}}(A \cup B)$ …… (2)
from (1) and (2) we get $Mic - int_{\dot{b}}(A) \cup Mic - int_{\dot{b}}(B) \subset Mic - int_{\dot{b}}(A \cup B)$.
2.since $A \cap B \subseteq A$ then $Mic - int_{\dot{b}}(A \cap B) \subseteq Mic - int_{\dot{b}}(A)$ and $A \cap B \subseteq B$ then $Mic - int_{\dot{b}}(A \cap B) \subseteq Mic - int_{\dot{b}}(B)$.
That is $Mic - int_{\dot{b}}(A \cap B) \subseteq Mic - int_{\dot{b}}(A) \cap Mic - int_{\dot{b}}(B)$ …… (1)
Now $Mic - int_{\dot{b}}(A) \subseteq A$ and $Mic - int_{\dot{b}}(B) \subseteq B$ therefore $Mic - int_{\dot{b}}(A) \cap Mic - int_{\dot{b}}(B) \subseteq A \cap B$.
That is $Mic - int_{\dot{b}}[Mic - int_{\dot{b}}(Mic - int_{\dot{b}}(A) \cap Mic - int_{\dot{b}}(B)] \subseteq Mic - int_{\dot{b}}(A \cap B)$.
Since $Mic - int_{\dot{b}}[Mic - int_{\dot{b}}(A) \cap Mic - int_{\dot{b}}(B)] = [Mic - int_{\dot{b}}(A) \cap Mic - int_{\dot{b}}(B)$.
Therefore $[Mic - int_{\dot{b}}(A) \cap Mic - int_{\dot{b}}(B)] \subseteq Mic - int_{\dot{b}}(A \cap B)$ …… (2)
From (1) and (2) we get $Mic - int_{\dot{b}}(A) \cap Mic - int_{\dot{b}}(B)] = Mic - int_{\dot{b}}(A \cap B)$
By reverse the steps we get to the converse □

Remark 3.27:
1. Intersection of Mic – α – open set and mic $-\dot{b}$ – open set is Mic $-\dot{b}$ – open set.
2. Union of Mic – semi open and Mic – pre – open sets is Mic $-\dot{b}$ – open set.
3. Union of Mic – pre – open and Mic $-\dot{b}$ – open is Mic $-\dot{b}$ – open set.
4. Union of Mic – semi – open and Mic $-\dot{b}$ $-$ open is Mic $-\dot{b}$ – open set.

4 Micro $-\dot{b}$ $-$ continuity

'In this section we introduced a new class of functions, namely micro $-b^{\cdot}$ $-$continuous briefly ($Mic - \dot{b} - continuous$) function in micro topological spaces and study some characterization of it and so introduced micro $-\dot{b}$ $-$irresolute continuous function briefly ($Mic - \dot{b} - i - continuous$) functions and it's decomposition with Mic $-\dot{b}$ $-$ continuous function.

Definition 4.1:

Let $(U, T_R(X), \mu_R(X))$ and $(V, \dot{T}_R(X), \dot{\mu}_R(X))$ be two Micro_topological spaces, then $f: (U, T_R(X), \mu_R(X)) \to (V, \dot{T}_R(X), \dot{\mu}_R(X))$ called Mic $-\dot{b}-$ continuous function $if\ f^{-1}(v)$ is Mic$-\dot{b}$ $-$open in U when ever v is Mic – open in V.

Example 4.2:

$U = \{p, q, r, s, t\}, U\backslash R = \{\{p\}, \{q\}, \{r, s, t\}\}\ and\ X = \{p, t\}$

Then $T_R(X) = \{U, \emptyset, \{p\}, \{p, r, s, t\}, \{r, s, t\}\}\ then\ \mu = \{t\}$

Then $Micro - O = \mu_R(X) = \{U, \emptyset, \{p\}, \{t\}, \{p, t\}, \{p, r, s, t\}, \{r, s, t\}\}$.

$V = \{1, 2, 3, 4, 5\}\ \ and\ V\backslash R = \{\{1\}, \{2, 3, 4\}, \{5\}\}$

$X = \{1, 2\} \subseteq U, then\ T`_R(X) = \{U, \emptyset, \{1\}, \{1, 2, 3, 4\}, \{2, 3, 4\}\}$

$If\ \mu = \{4\}$

Then $\mu`_R(X) = \{U, \emptyset, \{1], \{4\}, \{1, 4\}, \{1, 2, 3, 4\}, \{2, 3, 4\}\}$.

And the function is defined by $f(p) = 1, f(q) = 5, f(r) = 2, f(s) = 3\ then\ f\ is\ Mic - \dot{b} - continous\ \ since\ H = \{2, 3, 4\}\ is\ Mic - open\ set\ in\ V\ and\ f^{-1}(H) = \{r, s, t\}\ is\ Mic - \dot{b}$ $-$ open set in U.

Theorem 4.3:

Every Mic$-$ continuous function is Mic $-\dot{b}-$ continuous function.

Proof:

Let $f: U \to V$ be Mic – continuous function that is $f^{-1}(H)$ is Mic - open in U, when ever H is Mic - open in V, Then by (theorem 3.3) and hence $f^{-1}(V)$ is Mic -open in V

Hence $f: U \to V$ be Mic $-\dot{b}-$ continuous function □

The following theorem we establish a characterization of Mic $-\dot{b}$ $-$continuous functions in terms of Mic $-\dot{b}$ $-$ closure.

Theorem 4.4:

A function $f : (U, T_R(X), \mu_R(X)) \to (V, \dot{T}_R(X), \dot{\mu}_R(X))$ is Mic $-\dot{b}-$ continuous if and only if $f(Mic - cl_b(A)) \subseteq Mic - cl_b(f(A))$ for every subset of U.

Proof:

Let f be Mic $-\dot{b}$ $-$ continuous and $A \subseteq U$. Then $f(A) \subseteq V, Mic - cl_b\big(f(A)\big)$is Mic $-\dot{b}$ $-$ closed in V. since f is Mic$-\dot{b}$ $-$continuous, $f^{-1}(Mic - cl_b(f(A))$ is micro closed in U. $Since\ f(A) Mic - cl_b\big(f(A)\big),$ $A \subseteq f^{-1}\Big(Mic - cl_b\big(f(A)\big)\Big).\ Thus\ f^{-1}(Mic - cl_b(f(A))$ is Mic$-\dot{b}-$ closed set containing A. Therefore $Mic - cl_b(A) \subseteq f^{-1}(Mic - cl_b(A))$, That is $f(Mic\ - cl_b(A)) \subseteq Mic - cl_b(f(A))$.

Conversely, let $f\Big(Mic - cl_b(A)\Big) \subseteq Mic - cl_b\big(f(A)\big)$ for every subset A of U. If F is Mic $-\dot{b}$ $-$ closed in V,since$f^{-1}(F) \subseteq U, f(Mic - cl_b(f^{-1}(F))) \subseteq Mic - cl_b(f(f^{-1}(F)) \subseteq Mic - cl_b(F).that\ is, Mic - cl_b(f^{-1}(F) \subseteq f^{-1}(Mic - cl_b(F)).Therefore, Mic - cl_b(f^{-1}(F)) = f^{-1}(F).Therefore, f^{-1}(F)\ is\ micro\ closed\ in\ U\ for\ every\ Mic - \dot{b} - closed\ set\ F\ in\ V.That\ is\ f\ is\ Mic - \dot{b} - continous\ function$ □

Now we characterize Mic$-\dot{b}$ $-$ continuous functions in terms of inverse image of micro closure.

Theorem 4.5:

A function $f: (U, T_R(X), \mu_R(X)) \to (V, \dot{T}_R(X), \dot{\mu}_R(X))$ is Mic $-\dot{b}$ $-$continuous if and only if $Mic - cl_b\big(f^{-1}(B)\big) \subseteq f^{-1}(Mic - cl_b\ (B))$ for every subset B of V.

Proof :

Let f be Mic$-\dot{b}$ $-$ continuous and $B \in V$, $Mic - cl_b(B)$ is Mic$-\dot{b}$ $-$ closed in V and hence $f^{-1}\Big(Mic - cl_b(B)\Big)$ is a micro closed in U. Therefore,

$Mic\ [cl_b(f^{-1}Mic - cl_b(B))] = f^{-1}Mic - cl_b(B)).\ Since\ B \subseteq Mic - cl_b(B), f^{-1}(B) \subseteq f^{-1}(Mic - cl_b(B)).\ Therefore, Mic - cl_b(f^{-1}(B)) \subseteq Mic - cl_b(f^{-1}(Mic - cl_b(B))) = f^{-1}(Mic - cl_b(B)).That\ is, Mic - cl_b(f^{-1}(B)) \subseteq f^{-1}\ (f^{-1}(Mic - cl_b(B)))\ for\ every\ B \in V.Let\ B\ Mic - \dot{b} - closed\ in\ V.Then\ Mic - cl_b(B) = B.$
$By\ assumption,\ Mic - cl_b f^{-1}(B) \subseteq f^{-1}(Mic - cl_b(B)) = f^{-1}(B).Thus, Mic - cl_b(f^{-1}(B)) \subseteq f^{-1}(B).$ That is, $f^{-1}(B)$ is a micro closed set in U for every Mic $-\dot{b}$ $-$closed set $B\ in\ V$. Therefore, f is Mic$-\dot{b}$ $-$ continuous on U □

Definition 4.6:

Let $(U, T_R(X), \mu_R(X))$ and $(V, \dot{T}_R(X), \dot{\mu}_R(X))$ be two Micro topological spaces, then $f : (U, T_R(X), \mu_R(X)) \to (V, \dot{T}_R(X), \dot{\mu}_R(X))$ is Micro $-\dot{b}$ irresolute continuous function briefly (Mic$-\dot{b}$ $-i$ $-$continuous) $if\ f^{-1}(v)$is Mic$-\dot{b}-$open set in U whenever v is Mic $-\dot{b}-$open set in V .

Theorem 4.7:

Every Mic – continuous function is Mic $-\dot{b}$ $-i-$ continuous function.

Proof:

Let $f: U \to V$ be Mic – continuous. Let $H \subset V$ and h be a micro open set in V by (theorem 3.3) and since (f is Mic- continuous). $f^{-1}(H)$ is Mic$-\dot{b}$ $-$openset, then f is Mic$-\dot{b}-i-$ continuous

□

Theorem 4.8:

Let $(U, \mathrm{T}_R(X), \mu_R(X))$ and $(V, \dot{\mathrm{T}}_R(X), \dot{\mu}_R(X))$ and $(W, \ddot{\mathrm{T}}_R(X), \ddot{\mu}_R(X)))$ be three Micro topological spaces if $f: U \to V$ is Micro $-\dot{b}-$ continuous, and $g: V \to W$ is Micro$-\dot{b}-i-$ continuous then $gof : U \to W$ is Mic$-\dot{b}-$ continuous.

Proof:

Let G be Mic$-\dot{b}-$ open set in W, since g is Micro $-\dot{b}-i-$ continuous ,then $g^{-1}(G)$ is Mic $-\dot{b}$ $-$openset in V.

Now $(gof)^{-1}(G) = (f^{-1}og^{-1})(G) = f^{-1}o\,(g^{-1}(G))$,$Take$ $g^{-1}(G) = H$ which is Mic $-\dot{b}$ $-$open in V, Since f is Mic$-\dot{b}-$ continuous then $f^{-1}(H)$ is Mic - open in U, Hence $gof: U \to W$ is Micro $-\dot{b}-$continuous function □

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حول المجموعات المايكروية المفتوحة من النمط $\dot{b}$ و الأستمرارية المايكروية من النمط $\dot{b}$

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الملخص

الغرض من هذا البحث هو تعميم لمفهوم المجموعات المايكروية المفتوحة من النمط $\dot{b}$ وكذلك درسنا علاقتها مع بعض المجموعات المفتوحة الأخرى مثل المجموعة المفتوحة القبلية و المجموعة شبه المفتوحة والمجموعة المفتوحة من النمط ألفا والمجموعة المفتوحة من النمط بيتا كما قمنا بتعميم الأستمرارية المايكروية من النمط $\dot{b}$ والأستمرارية المايكروية المترددة من النمط $\dot{b}$ و دراسة العلاقة فيما بينهما و لقد قدمنا الكثير من المبرهنات و الخصائص لهذه الأنواع من الأستمرارية مع بعض الأمثلة .