IASJ
المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j

On stability Conditions of Pareto Autoregressive model

Osama A. Hamdi , Azher A. Mohammad , Mundher A. Khaleel

Mathematics Department , College of Computer Science and Mathematics , Tikrit University , Tikrit , Iraq

https://doi.org/10.25130/tjps.v25i5.297

ARTICLE INFO.

Article history:

-Received: 5 / 6 / 2020

-Accepted: 22 / 7 / 2020

-Available online: / / 2020

Keywords: Pareto autoregressive model, Non-linear time series model, Stability, Local linearization Method, Limit cycle,

Corresponding Author:

Name: **Osama A. Hamdi**

E-mail:

adamant.aldoury96@gmail.com

Tel:

ABSTRACT

This paper concerned with studding a stability conditions of the proposed non-linear autoregressive time series model Known as Pareto Autoregressive model, acronym is defined by Pareto $AR\ (p)$. A dynamical method Known as local linearization approximation method was used to obtain the stability condition of a non-zero singular point of Pareto $AR\ (p)$ model. In addition, we obtain the orbital stability condition of a limit cycle in terms of model parameters when the Pareto AR (1) possesses a limit cycle with period $q > 1$.

1- Introduction

The nonlinear nature of most physical phenomena such as random vibrations and nonlinear oscillation necessitates that mathematical models have nonlinearity in order to be closer to reality and to give more accurate explanations that linear models additional to the fact that these models clearly show the characteristics of this nonlinear nature and the most important nonlinear characteristics are.

1. Jump phenomena that are evident in your burial in Duffing equations.

$\ddot{x}(t) + b\dot{x}(t) + cx(t) + dx^3(t) = F\cos vt$ ... (1.1)

where b, c, d are real constants, factor, b $\dot{x}$(t) is the damping force, $F\cos(vt)$ be the external force and cx(t)+dx^3 (t) is the restoring force

2. Reliability between wave amplitude and frequency which is clearly shown by the Doffing equation

3. The presence of the limit cycle behavior and This behavior is characterized by the following Vander - Pol equation

$\ddot{x}(t) - d.[1 - x^2(t)].\dot{x}(t) + c.x(t) = 0$ ... (1.2)

A local linear approximation method used to approximate nonlinear differential equations to linear differential equation near the fixed point of the system gives almost better than the straight segment approximation method, especially if the solution is cyclic to clarify the approximate method of local linearity, we consider the non – linear differential equation, which is the following Vander- Pol equation

$\ddot{x} + (x^2 - 1).\dot{x} + x = 0$ … (1.3)

By setting $\dot{x} = y$ and $\dot{y} = \ddot{x}$ equation (1.3) will become a system consisting of the following two differential equations

$\dot{x} = y$ ... (1.4a)

$\dot{y} = (1 - x^2).y - x$ ... (1.4b)

System (1.4a) , (1.4b) is called a two – dimensional system the state space and the solution is the vector $(x, y)^T$ which is called the path trajectory.

Taking the first and second terms of Tylor expansion of (1.4) around the origin we get

$\dot{X} = X\,(0,0) + \left[\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right] y$

$\dot{Y} = Y\,(0,0) + \left[\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right]((1 - x^2).y - x)$

Note that both X and Y are functions of x and y , that is $X = X(x, y)$ and $Y = Y(x, y)$, and $X\,(0,0) = 0, Y(0,0) = 0$, then the system becomes

$\dot{x} = y$

$\dot{y} = (-1 - 2xy) + (1 - x^2)$

Or in Matrix form

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1-2xy & 1-x^2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = L(x,y)\begin{bmatrix}x\\ y\end{bmatrix}$$

Then a local linearization approximation of the system (1.3) near the fixed point gives

$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = L(0,0)\begin{bmatrix}x\\ y\end{bmatrix}$ or $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$

That is mean

$\dot{x} = y \quad , \dot{y} = -x + y$ … (1.5)

And then we get

$\ddot{x} - \dot{x} + x = 0$ … (1.6)

In fact, the origin point is not the only fixed point of the system (1,2) thus, Chen has stated that the method of approximation to local linearity can be applied to non – Zero fixed points. [1]

The local linearization technique can be applied to approximate a nonlinear autoregressive time series model to a linear one and then obtaining the stability conditions of a model in terms of its parameters. Many researchers were used this technique such as Ozaki T. in 1982 and in 1985 used this dynamical technique in studding the exponential autoregressive model.[2],[3], in 2007 Mohammad and Salim In studding logistic Autoregressive model.[4], In 2010 Mohammed and Ghannam used this technique in studding Cauchy autoregressive model.[5], Salim and Esmaeel and Jasim in 2011 used the same method In studding stability of amplitude dependent exponential autoregressive model.[6], in 2012 Salim and Younis in studding the stability of a non-linear autoregressive models with trigonometric function.[7], Salim and Abdullah in 2014 studied the stability conditions of a polynomial model with hyperbolic cosine function. [8], , Mohammad and Ghaffar in 2016 in studying stability of conditional variance GARCH models.[9], Mohammad and Mudhir in 2020 used the local linearization technique in studding the stability condition of exponential GARCH model by a dynamical approach [10] .

2- Preliminaries

Pareto probability distribution function depend into two parameters, the first one is the minimum value of a random variable X denoted by x_m and called the scale parameter, the second parameter a be the shape parameter and the cumulative distribution function (c.d.f) of Pareto distribution given by

$F(X,a,x_m) = 1 - \left(\frac{x_m}{X}\right)^a \quad , X \geq x_m , x_m > 0 , a > 0$ …(2.1)

The probability density function (p.d.f) of Pareto distribution is

$f(X,a,x_m) = \frac{a\, x_m^a}{X^{a+1}} \quad , X \geq x_m , x_m > 0 , a > 0$

Figure (2.1) represents the graph of the cumulative distribution function of the Pareto distribution with the values of a. . [11], [12]

The smooth jump from 0 to 1 in the graph of cumulative distribution function characterized the nonlinear behavior of this function , then it's useful to define and suggest the Pareto autoregressive model which is one of nonlinear time series

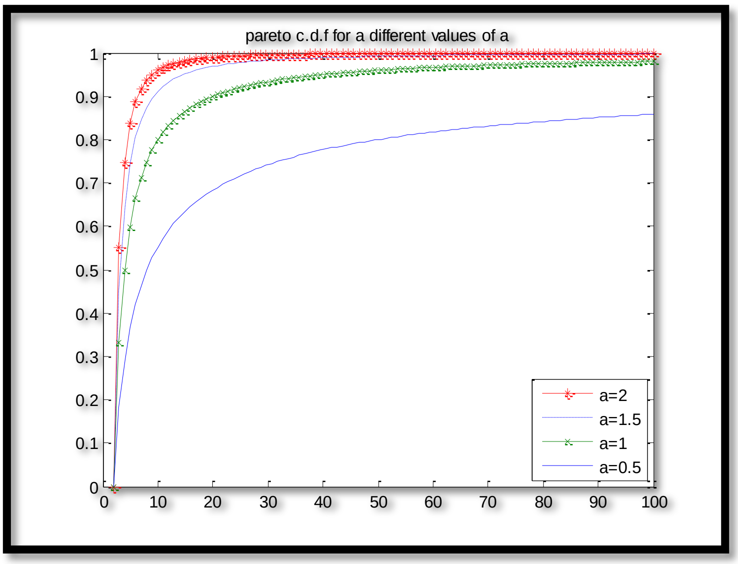

Fig 2.1: graph of Pareto c.d.f with different values of a

Definition 2.1

Let $\{x_t\}$ be a discrete time series then the Pareto AR (p) model is defined as follows :

$X_t = \sum_{l=1}^{p}\left[\alpha_l + \beta_l\{1 - (\frac{x_m}{X_{t-1}})^a\}\right] . X_{t-l} + Z_t \quad , \quad Z_t \sim iidN(0, \sigma_Z^2)$ … (2.2)

Where $\{z_t\}$ be a white noise process, a and x_m are shape and scale parameters, $\{\alpha_l\}$ and $\{\beta_l\}, l = 1,2,3, \dots . p$ are constants.

Definition 2.2

Let T be a finite positive integer. A k-dimensional vector $\mathbf{Y}^*$ is called periodic point with period T if $\mathbf{Y}^* = f^T(\mathbf{Y}^*)$ and $\mathbf{Y}^* \neq f^j(\mathbf{Y}^*)$ for $1 \leq j < T$

Here $\mathbf{Y}^*$ is a fixed point of f^T, we say that $\mathbf{Y}^*$ is a periodic point with period T for some T≥1. And the ordered set $\{\mathbf{Y}^*, f(\mathbf{Y}^*), f^2(\mathbf{Y}^*), \dots, f^{T-1}(\mathbf{Y}^*)\}$ is called a T-cycle. We say that $\mathbf{Y}_0$ is eventually periodic if there is a positive integer n such that $\mathbf{Y}^* = f^n(\mathbf{Y}_0)$ is periodic. We say that $\mathbf{Y}_0$ is asymptotically periodic if there exists periodic point $\mathbf{Y}^*$ for which $\|f^n(\mathbf{Y}_0) - f^T(\mathbf{Y}^*)\| \to 0 \quad as \ n \to \infty$. [1]

Definition 2.3

A singular point r of a model $X_T = f(X_{T-1}, X_{T-2}, \dots, X_{T-p})$ where f is linear or nonlinear function is defined to be a point for which every trajectory of the model beginning sufficiently closed to the singular point r approaches to it either for $T \to \infty$ or $T \to -\infty$. If it approaches to a singular point for $T \to \infty$, then its stable singular point, and if it approaches to a singular point for $T \to -\infty$, then its unstable singular point.[13], [14]

Definition 2.4

A limit cycle of a model $X_T = f(X_{T-1}, X_{T-2}, \dots, X_{T-p})$ where f is nonlinear function is defined as an closed isolated trajectory

$y_T, y_{T+1}, y_{T+2}, \dots, y_{T+q} = y_T$ ... (2.3)

Where the period $q > 1$ be a smallest positive integer such that $y_{T+q} = y_T$. Closed means that if the initial value $(y_1, y_2, y_3, \dots, y_p)$ belongs to the limit cycle, then$(y_{1+kq}, y_{2+kq}, \dots, y_{p+kq}) = (y_1, y_2, y_3, \dots, y_p)$ for any $k \in Z^+$. By Isolated we mean that every trajectory being sufficiently closed to the limit cycle approaches to it for $T \to \infty$ or $T \to -\infty$ If it approaches to the limit cycle for $T \to \infty$, then the limit cycle is , but if it approaches to the limit cycle for $T \to -\infty$, then the limit cycle is unstable. [13], [14]

3- Stability condition Pareto AR(p) model

In this paragraph we study stability for the time series model (2.2)

Whereas, the function $\left[1-\left(\frac{x_m}{x_{t-1}}\right)^a\right]$ has the following properties

1- $\lim_{x_{t-1}\to x_m} F(x_{t-1}, a, x_m) = 1$

2- $\lim_{x_{t-1}\to \mp\infty} F(x_{t-1}, a, x_m) = 0$

The characteristic equation of the model (2.2) represent the transition between the two Scheme $\sum_{l=1}^{p}(\alpha_l + \beta_l)$ when $x_{t-1} \to x_m$ and $\sum_{l=1}^{p}\alpha_l$ when $x_{t-1} \to \mp\infty$

To study the stability conditions of (2,2) by using a local linearization technique , we first find the nonzero singular point of the model by putting $x_{t-s} = r$ for $s = 0,1,2,\dots,p$ in (2,2) and suppressing a white noise process Z_t , we get

$$r=\sum_{l=1}^{p}\left[\alpha_l + \beta_l\left(1-\left(\frac{x_m}{r}\right)^a\right)\right] r$$

$$1-\sum_{l=1}^{p}\alpha_l = \sum_{l=1}^{p}\beta_l - \sum_{l=1}^{p}\beta_l\left(\frac{x_m}{r}\right)^a$$

$$1-\left(\sum_{l=1}^{p}(\alpha_l+\beta_l)\right) = -\sum_{l=1}^{p}\beta_l\left(\frac{x_m}{r}\right)^a$$

$$\frac{1-\left(\sum_{l=1}^{p}(\alpha_l+\beta_l)\right)}{-\sum_{l=1}^{p}\beta_l} = \left(\frac{x_m}{r}\right)^a$$

$$\frac{\left(\sum_{l=1}^{p}(\alpha_l+\beta_l)\right)-1}{\sum_{l=1}^{p}\beta_l} = \left(\frac{x_m}{r}\right)^a$$

Let $k = \frac{\left(\sum_{l=1}^{p}(\alpha_l+\beta_l)\right)-1}{\sum_{l=1}^{p}B_l}$ then $k^{\frac{1}{a}} = \frac{x_m}{r}$ and we get

$$r = \frac{x_m}{\sqrt[a]{k}} \quad \dots (3.1)$$

Then the non-zero singular point for Pareto AR (p) model exists and real if $k > 0$, that is

$$\frac{\left(\sum_{l=1}^{p}(\alpha_l+\beta_l)\right)-1}{\sum_{l=1}^{p}B_l} > 0 \quad \dots (3.2)$$

PROPOSITION 3.1

Pareto AR (p) model defined in (2, 2) is asymptotically stable if all the roots of characteristic equation

$\lambda^p - \sum_{j=1}^{p} h_j \lambda^{p-j} = 0$ lies inside the unit circle, where

$$h_1 = \alpha_1 + \beta_1(1-k) + ak\sum_{l=1}^{p}\beta_l \quad \dots (3.3)$$

$$h_j = \alpha_j + (1-k)\beta_j \quad \text{for } j = 1,2,3,\dots,p \quad \dots (3.4)$$

Proof:

Closed to the nonzero singular point of the model we consider the variation difference equation in the neighborhood of r by setting $x_{t-s} = r + r_{t-s}$, where r_{t-s} be the radius of a neighborhood such that $|r_{t-s}|^n \to 0$ for $n \geq 2$, $s = 0,1,2,\dots,p$ in Pareto AR(p) model (2.2) after the white noise be suppressed we get

$$r + r_t = \sum_{l=1}^{p}\left[\alpha_l + \beta_l\left(1-\left(\frac{x_m}{r+r_{t-1}}\right)^a\right)\right](r + r_{t-l}) \quad \dots (3.5)$$

By using Maclaurin expansion of $\left(1+\frac{r_{t-1}}{r}\right)^{-a}$ we get

$$1-\left(\frac{x_m}{r+r_{t-1}}\right)^a = 1-\left(\frac{r+r_{t-1}}{x_m}\right)^{-a}$$

$$1-\left(\frac{x_m}{r+r_{t-1}}\right)^a = 1-\left(\left(\frac{1}{x_m}\right)^{-a}\right)(r)^{-a}\left(1+\frac{r_{t-1}}{r}\right)^{-a}$$

$$= 1-\left(\left(\frac{x_m}{r}\right)^a\right)\left(1-\frac{a\, r_{t+1}}{r} + (-a)(-a-1)\frac{r_{t-1}^2}{r} + \cdots\right)$$

But $r_{t-1}^2 \to 0$ then

$$1-\left(\frac{x_m}{r+r_{t-1}}\right)^a = 1-\left(\frac{x_m}{r}\right)^a\left(1-\frac{a r_{t-1}}{r}\right)$$

$$= 1-k\left(1-\frac{a r_{t-1}}{\frac{x_m}{\sqrt[a]{k}}}\right) = 1-k\left(1-\frac{a\sqrt[a]{k}\, r_{t-1}}{x_m}\right) \text{, then}$$

$$1-\left(\frac{x_m}{r+r_{t-1}}\right)^a = 1-k+\left(\frac{ak\sqrt[a]{k}\, r_{t-1}}{x_m}\right) \dots (3.6)$$

And by substituting (3.6) in equation (3.5) we get

$$r + r_t = \sum_{l=1}^{p}\left[\alpha_i + \beta_i\left(1-k+\left(\frac{ak\sqrt[a]{k}\, r_{t-1}}{x_m}\right)\right)\right](r + r_{t-l})$$

$$r + r_t = \sum_{l=1}^{p}\left[\alpha_l + \beta_l - \beta_l k + \left(\frac{\beta_l ak\sqrt[a]{k}\, r_{t-1}}{x_m}\right)\right](r + r_{t-l})$$

$$r + r_t = \sum_{l=1}^{p}\alpha_l r + \sum_{l=1}^{p}\alpha_l r_{t-l} + \sum_{l=1}^{p}\beta_l r + \sum_{l=1}^{p}\beta_l r_{t-l} - \sum_{l=1}^{p}\beta_l kr - \sum_{i=1}^{p}\beta_l k r_{t-l} + \frac{\sum_{l=1}^{p}\beta_l ak\sqrt[a]{k}\, r_{t-1} r}{x_m} + \frac{\sum_{l=1}^{p}\beta_l ak\sqrt[a]{k}\, r_{t-1} r_{t-l}}{x_m}$$

But $|r_{t-1} r_{t-l}| \to 0$ then

$$r + r_t = r\left[\sum_{l=1}^{p}\alpha_l + (1-k)\sum_{l=1}^{p}\beta_l\right] + \frac{\sum_{l=1}^{p}\beta_l ak\sqrt[a]{k}\, r_{t-1} r}{x_m} + \sum_{l=1}^{p}[\alpha_l + \beta_l(1-k)]r_{t-1}$$

$$r_t = \frac{\sum_{l=1}^{p}\beta_l ak\sqrt[a]{k}\, r_{t-1}}{x_m}\frac{x_m}{\sqrt[a]{k}} + \sum_{l=1}^{p}\alpha_l + (1-k)\beta_l r_{t-1}$$

$$r_t = \left[\alpha_1 + (1-k)\beta_1 + ak\sum_{l=1}^{p}\beta_l\right]r_{t-1} + \sum_{l=2}^{p}[\alpha_l + \beta_l(1-k)]r_{t-l} \quad \dots (3.7)$$

which is a linear difference equation of order p in the form

$r_t = h_1 r_{t-1} + h_2 r_{t-2} + \cdots h_p r_{t-p}$ … (3.8)

where

$h_1 = \alpha_1 + \beta_1(1-k) + \mathbf{a}k\sum_{l=1}^{p}\beta_l$

$h_j = \alpha_j + (1-k)\beta_j$ for $j = 1,2,3 \ldots\ldots p$

And the non-zero singular point of the Pareto AR (p) model is stable if all the roots of characteristic equation given by $\lambda^p - \sum_{j=1}^{p} h_j \lambda^{p-j} = 0$ lies inside unit circle

In the following proposition we find the stability condition of a limit cycle when Pareto AR (1) possess a limit cycle of period $q > 1$

PROPOSITION 3.2

If the following Pareto AR (1) model possess a limit cycle of period $q > 1$

$$x_t = \left[\alpha_1 + \beta_1\left(1 - \left(\frac{x_m}{x_{t-1}}\right)^a\right)\right] x_{t-1} + Z_t \quad \ldots (3.9)$$

Then the model (3.9) is orbital stable if

$$\left|\prod_{j=1}^{q}\left[\alpha_1 + \beta_1\left(1-(1-a)\left(\frac{x_m}{x_{t-1}}\right)^a\right)\right]\right| < 1 \quad \ldots (3.10)$$

Proof:

Let $x_t, x_{t+1}, x_{t+2}, \ldots, x_{t+q} = x_t$ be a limit cycle of period $q > 1$, near each point of a limit cycle x_s suppose r_s be the radius of a neighborhood whose center is the point x_s such that $r_s^n \to 0$ for $n \geq 2$ and for $s = t, t+1, \ldots, t+q$, by replacing x_s in (3.10) by $x_s + r_s$ for $s = t, t-1$ after suppressing a white noise process we get

$$x_t + r_t = \left[\alpha_1 + \beta_1\left(1 - \left(\frac{x_m}{x_{t-1} + r_{t-1}}\right)^a\right)\right](x_{t-1} + r_{t-1}) \quad \ldots (3.11)$$

By using Maclaurin expansion of $1 - \left(\frac{x_m}{x_{t-1}+r_{t-1}}\right)^a$ we get

$$1 - \left(\frac{x_m}{x_{t-1}+r_{t-1}}\right)^a = 1 - \left(\frac{x_{t-1}+r_{t-1}}{x_m}\right)^{-a}$$

$$= 1 - \left(\frac{1}{x_m}\right)^{-a}(x_{t-1})^{-a}\left(1 + \frac{r_{t-1}}{x_{t-1}}\right)^{-a}$$

$$= 1 - \left(\frac{x_m}{x_{t-1}}\right)^a\left(1 - \frac{a\, r_{t-1}}{x_{t-1}}\right), \text{ then}$$

$$1 - \left(\frac{x_m}{x_{t-1}+r_{t-1}}\right)^a = 1 - \left(\frac{x_m}{x_{t-1}}\right)^a + \left(\frac{x_m}{x_{t-1}}\right)^a \frac{a\, r_{t-1}}{x_{t-1}} \quad \ldots (3.12)$$

By substituting (3.12) in equation (3.11) we get

$$x_t + r_t = \left[\alpha_1 + \beta_1\left(1 - \left(\frac{x_m}{x_{t-1}}\right)^a + \left(\frac{x_m}{x_{t-1}}\right)^a \frac{a r_{t-1}}{x_{t-1}}\right)\right](x_{t-1} + r_{t-1})$$

$$x_t + r_t = \alpha_1 x_{t-1} + \alpha_1 r_{t-1} + \beta_1 x_{t-1}\left(1 - \left(\frac{x_m}{x_{t-1}}\right)^a + \left(\frac{x_m}{x_{t-1}}\right)^a \frac{a r_{t-1}}{x_{t-1}}\right.$$

$$\left. + \beta_1 r_{t-1}\left(1 - \left(\frac{x_m}{x_{t-1}}\right)^a + \left(\frac{x_m}{x_{t-1}}\right)^a \frac{a r_{t-1}}{x_{t-1}}\right)\right)$$

$$x_t + r_t = \left(\alpha_1 + \beta_1\left(1 - \left(\frac{x_m}{x_{t-1}}\right)^a\right)\right)x_{t-1} + \beta_1 x_{t-1}\left(\frac{x_m}{x_{t-1}}\right)^a \frac{a r_{t-1}}{x_{t-1}}$$

$$+ \alpha_1 r_{t-1} + \beta_1 r_{t-1} - \beta_1 r_{t-1}\left(\frac{x_m}{x_{t-1}}\right)^a + \beta_1\left(\frac{x_m}{x_{t-1}}\right)^a \frac{a\, r_{t-1}^2}{x_{t-1}}$$

But $r_{t-1}^2 \to 0$ then

$$r_t = \beta_1\left(\frac{x_m}{x_{t-1}}\right)^a a\, r_{t-1} + \beta_1 r_{t-1} - \beta_1 r_{t-1}\left(\frac{x_m}{x_{t-1}}\right)^a + \alpha_1 r_{t-1}$$

$$r_t = \left[\alpha_1 + \beta_1\left(1 - (1-a)\left(\frac{x_m}{x_{t-1}}\right)^a\right)\right] r_{t-1} \quad \ldots (3.13)$$

equation (3.13) is difference equation of variable coefficients of the first order and it's difficult to solve exactly but we discuss the convergence of the equation (3.13) to zero by checking the ratio $\left|\frac{r_t}{r_{t+q}}\right|$, the difference equation (3.13) is stable $(\lim_{t\to\infty} r_t = 0)$ if $\left|\frac{r_t}{r_{t+q}}\right| < 1$

Let $T(x_{t-1}) = \alpha_1 + \beta_1\left(1 - (1-a)\left(\frac{x_m}{x_{t-1}}\right)^a\right)$, then we can write

$r_{t+1} = T(x_t) r_t$

Consequently

$r_{t+q} = T(x_{t+q-1}) r_{t+q-1} = T(x_{t+q-1}) T(x_{t+q-2}) r_{t+q-2} = T(x_{t+q-1}) T(x_{t+q-2}) T(x_{t+q-3}) r_{t+q-3}$

And after q iteration we get

$r_{t+q} = \prod_{j=1}^{q} T(x_{t+q-j}) . r_t$, then

$\left|\frac{r_{t+q}}{r_t}\right| = \left|\prod_{j=1}^{q} T(x_{t+q-j})\right|$

Finally the difference equation (3.13) is stable if

$\left|\prod_{j=1}^{q} T(x_{t+q-j})\right| < 1$

Finally, the limit cycle (if it exists) of Pareto AR (1) model is orbitally stable if

$$\left|\prod_{j=1}^{q}\left[\alpha_1 + \beta_1\left(1 - (1-a)\left(\frac{x_m}{x_{t-1}}\right)^a\right)\right]\right| < 1$$

Example 3.1

Consider the following Pareto AR (3) model that obtained by modelling the monthly mean Ozone data for the years (1990-2019)

$$x_t = \sum_{i=1}^{3}\left[\alpha_i + \beta i\left(1 - \left(\frac{x_m}{x_{t-1}}\right)^a\right)\right] x_{t-i} + z_t \ldots (3.15)$$

Where the parameter's values are given by

$\alpha_1 = 1.571292$, $\beta_1 = -0.178009$

$\alpha_2 = -0.582187$, $\beta_2 = -0.337623$

$\alpha_3 = 0.066526$, $\beta_3 = 0.082280$

$x_m = 217.035$, $a = 0.772381$

By using (3.2) we calculate *k=0.8715 ≥0* , then the nonzero singular point exists and equal to 259.2701 and by applying equations (3.3) and (3.4) we obtain that h_1 = 1.2567 , h_2 = -0.6256 , h_3 = 0.0771 , and the root of the characteristic equation

$\lambda^3 - 1.12567\lambda^2 + 0.6256\lambda - 0.0771 = 0$ are

$\lambda_1 = 0.17777, \lambda_2 = 0.53946 + 0.37772\, i, \lambda_3 = 0.53946 - 0.37772i$

Since $|\lambda_1| = |0.17777| < 1$ and

$|\lambda_2| = |\lambda_3| = \sqrt{(053946)^2 + (0.37771)^2} = \sqrt{0.4337} = 0.6586 < 1$

Then the Model (3.15) is asymptotically stable. Fig (3.1) shows the convergence of trajectories that starting from different initial values to a nonzero singular point of the model.

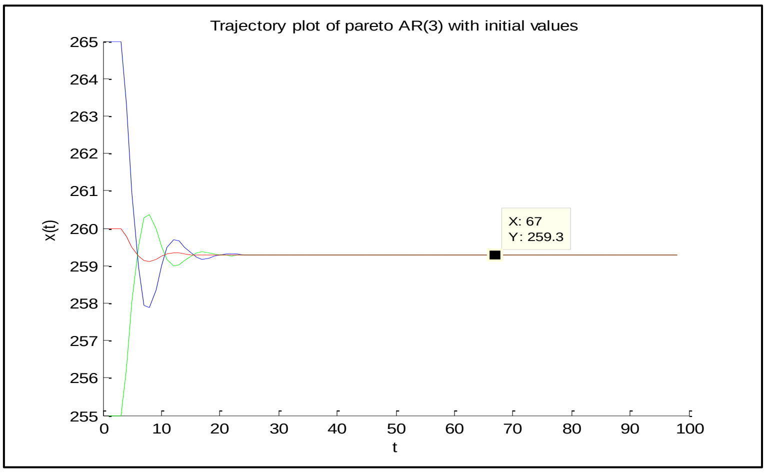

Fig. 3.1: Trajectory plot of Model (3.15)

References

[1] Tong, H.," Nonlinear Time Series. A Dynamical System Approach", Oxford University Press, New York, 1990

[2] Ozaki, T, "The Statistical Analysis of Perturbed Limit Cycle Processes Using Nonlinear Time Series Models", Journal of Time Series Analysis, Vol.3, N0.1, PP (29-41), 1982

[3] Ozaki, T., "Nonlinear Time Series Models and Dynamical Systems", Handbook of Statistics, Vol. 5, Hannan, E. J. and Krishnailah, P.R. and Rao, M. M., Elsevier Science Publishers R V., PP (25- 83), 1985.

[4] Mohammad A.A., Salim A.J, "Stability of Logistic Autoregressive Model ", Qatar university of Science journal, Vol (27), PP (17-28), 2007.

[5] Mohammad, A. A., Gannam, A. K., "Stability of Cauchy autoregressive model", Zanco journal of Pure and Applied Sci. – Sallahaddin University – Hawler (special Issue), 2010.

[6] Salim A. J., Esmaeel k., Jasim H. T., "A study of the stability of an amplitude-dependent exponential autoregressive model with application", Iraqi Journal of Statistical Science, The Fourth Scientific Conference of the College of Computer Science & Mathematics pp (52-62),2011.

[7] Salim, A.J., Youns, A. S.," A Study of the Stability of a Non-Linear Autoregressive Models", Australian J. Basic Appl. Sci. 6 (13), pp (159–166), 2012.

[8] Salim, A. G. J., Abdullah, A. S. Y., " Studying the Stability of a Non-linear Autoregressive Model (Polynomial with Hyperbolic Cosine Function)", AL-Rafidain Journal of Computer Sciences and Mathematics, 11(1), pp (81-91), 2014.

[9] Mohammad A. A., Ghaffar M. K., "A study on stability of conditional variance for GARCH models with application", Tikrit journal of pure science, (4) 21,2016.

[10] Mohammad, A. A. and Mudhir, A. A.,"Dynamical approach in studying stability condition of exponential (GARCH) model ", journal of king saud university-science, issue 32, pp(272-278),2020.

[11] NORMANL SOHNSONM. “continuous univariate distributions”, university of North Carolina, second Edition, Vol (1), pp (573-620), 1995.

[12] Namah, M.W., “Estimating parameters Gumbel Pareto distribution”, DIYALA JOURNAL FOR PURE SCIENCES, 14(2), pp(53-60),2018.

[13] Ozaki, T.," Non-linear time series models for non-linear random vibrations", Journal of Applied Probability, 17(1), pp (84-93), 1980.

[14] Haggan, V., & Ozaki, T., "Modelling nonlinear random vibrations using an amplitude-dependent autoregressive time series model", Biometrika, 68(1), pp (189-196), 1981.

حول شروط استقرارية انموذج pareto للانحدار الذاتي

اسامة اسعد حمدي ، ازهر عباس محمد ، منذر عبدالله خليل

قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

الهدف من هذه الورقة هي دراسة شروط استقراريه نموذج المتسلسلات الزمنية الانحدار الذاتي الغير خطي المقترح والمسمى بنموذج باريتو للانحدار الذاتي من الرتبة p ويعرف اختصارا بـ ((Pareto AR(p)). تم استخدام طريقة التقريب الخطية المحلية لإيجاد شروط استقراريه النقطة المنفردة غير الصفرية للنموذج المقترح تبعا لمعلماته. اضافة الى ايجاد شروط الاستقرار المداري للغاية الدورية وفقا لمعلمات نموذج باريتو للانحدار الذاتي من الرتبة الاولى ((Pareto AR (1)) في حالة امتلاكه لغاية دورية بالدورة $q > 1$.