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Temperature and Multiplicative Temperature Indices of Nanotubes $TUC_4C_8[p,q]$

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ABSTRACT

Topological indices are a numerical parameter of a graph ,sometimes also called as a
graph –theoretic index, is a number invariant of a chemical graph .Particular topological indices include the types of temperature and multiplicative temperature indices such as the first and second hyper temperature , multiplicative first and second hyper temperature and etc.
In this paper we compute the many of types for temperature indices of $TUC_4C_8[p,q]$ nanotubes .

1. Introduction

Let G be a finite simple connected graph with vertex set $V(G)$ and edge set $E(G)$, the degree $d_G(u)$ of a vertex u is the number of edges adjacent to u [1].
Chemical Graph theory is a branch of mathematical chemistry which deals with the nontrivial appli-cations of graph theory to solve molecular problems. The Chemical Graph is used as a convenient model for any real or abstract chemical system(reaction scheme or molecule in chemical transform-ation), [2,3]. There are many topological indices in chemical graph like Wiener, Randic connecti-vity ,Harary, Atom bond connectivity, Szeged, arithmetic - geometric, geometric-arithmetic indi - ces, etc. In 1988 Fajtlowicz defined the temperature of a vertex v of a graph G as $T(v)=\frac{d_G(v)}{n-d_G(v)}$,where n is the number of vertices of G [4] .
Some temperature indices were studied in[5,6,7,8] and also some new connectivity indices were- studied in [9,10,11] in this paper some temperature indices and some multiplicative temperature indices of $TUC_4C_8[p,q]$ nanotube are computed .
In the following table, we will give definitions of the types of temperature indices of any graph G,[5,6,7,8].

Table 1.1

Type of temperature index	Definition or formula
Multiplicative first temperature index of G	$TII_1(G)=\prod_{uv\in E(G)}[T(u)+T(v)]$.
Multiplicative second temperature index of G	$TII_2(G)=\prod_{uv\in E(G)}[T(u)\,T(v)]$.
First hyper temperature index of G	$HT_1(G)=\sum_{uv\in E(G)}[T(u)+T(v)]^2$.
Second hyper temperature index of G	$HT_2(G)=\sum_{uv\in E(G)}[T(u)\,T(v)]^2$.
Multiplicative first hyper temperature index of G	$HTII_1(G)=\prod_{uv\in E(G)}[T(u)+T(v)]^2$.
Multiplicative second hyper temperature index of G	$HTII_2(G)=\prod_{uv\in E(G)}[T(u)\,T(v)]^2$.
General first hyper temperature index of G	$T_1^a(G)=\sum_{uv\in E(G)}[T(u)+T(v)]^a$,where a is a real number .
General second hyper temperature index of G	$T_2^a(G)=\sum_{uv\in E(G)}[T(u)\,T(v)]^a$,where a is a real number .
General multiplicative first hyper temperature index of G	$TII_1^a(G)=\prod_{uv\in E(G)}[T(u)+T(v)]^a$,where a is a real number .
General multiplicative second hyper temperature index of G	$TII_2^a(G)=\prod_{uv\in E(G)}[T(u)\,T(v)]^a$,where a is a real number .
F- temperature index of G	$FT(G)=\sum_{uv\in E(G)}[T(u)^2+T(v)^2]$
Multiplicative F- temperature index of G	$FTII(G)=\prod_{uv\in E(G)}[T(u)^2+T(v)^2]$.
General F- temperature index of G	$T_a(G)=\sum_{uv\in E(G)}[T(u)^a+T(v)^a]$, where a is a real number.
General multiplicative F-temperature index of G	$TII_a(G)=\prod_{uv\in E(G)}[T(u)^a+T(v)^a]$, where a is a real number.
Product connectivity temperature index of G	$PT(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{T(u)T(v)}}$.
Multiplicative product connectivity temperature index of G	$PTII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{T(u)T(v)}}$.
Reciprocal product connectivity temperature index of G	$RPT(G)=\sum_{uv\in E(G)}\sqrt{T(v)T(v)}$.
Multiplicative reciprocal product connectivity temperature index of G	$RPTII(G)=\prod_{uv\in E(G)}\sqrt{T(v)T(v)}$.
Sum connectivity temperature index of G	$ST(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{T(u)+T(v)}}$.
Multiplicative sum connectivity temperature index of G	$STII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{T(u)+T(v)}}$
Arithmetic-geometric temperature index of G	$AGT(G)=\sum_{uv\in E(G)}\frac{T(u)+T(v)}{2\sqrt{T(u)T(v)}}$.
Multiplicative arithmetic-geometric temperature index of G	$AGTII(G)=\prod_{uv\in E(G)}\frac{T(u)+T(v)}{2\sqrt{T(u)T(v)}}$.
Atom bond connectivity temperature index of G	$ABCT(G)=\sum_{uv\in E(G)}\sqrt{\frac{T(u)+T(v)-2}{T(u)T(v)}}$.
Multiplicative atom bond connectivity temperature index of G	$ABCTII(G)=\prod_{uv\in E(G)}\sqrt{\frac{T(u)+T(v)-2}{T(u)T(v)}}$.
Geometric- arithmetic temperature index of G	$GAT(G)=\sum_{uv\in E(G)}\frac{2\sqrt{T(u)T(v)}}{T(u)+T(v)}$.
Multiplicative geometric- arithmetic temperature index of G	$GATII(G)=\prod_{uv\in E(G)}\frac{2\sqrt{T(u)T(v)}}{T(u)+T(v)}$.

2. Main Results

We consider $TUC_4C_8[p,q]$ is an nanotubes, where p and q are denotes number of octagons in a fixed row and column respectively. Let $G=TUC_4C_8[p,q]$, by calculation G has $3pq(13-q)+q(7q-q^2-16)-24p+12$ number of edges,,see Figure 2. 1.[12].

There are seven types of edges based on the temperature of the vertices of each edge ,see Tabel 2.1 , It is easy to see that G has $10pq+q-p$ vertex, see Figure 2.1.

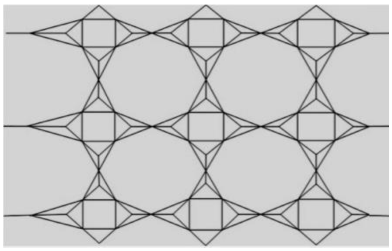

Fig. 2. 1 :Nanotube $TUC_4C_8[3,3]$

Table 2.1. :Edge partition of *G*.

(d_u,d_v)	Number of edges	$T(u),T(v)$, $u\,v\in E(G)$
(2,5)	$4p$	$\frac{2}{10pq+q-p-2},\frac{5}{10pq+q-p-5}$
(3,3)	$2q$	$\frac{3}{10pq+q-p-3},\frac{3}{10pq+q-p-3}$
(3,5)	$2pq(5-q)-4(2p-1)$	$\frac{3}{10pq+q-p-3},\frac{5}{10pq+q-p-5}$
(3,6)	$17pq-(16p+3q)+q^2(1-p)+2$	$\frac{3}{10pq+q-p-3},\frac{6}{10pq+q-p-6}$
(5,5)	$8p-5pq+pq^2$	$\frac{5}{10pq+q-p-5},\frac{5}{10pq+q-p-5}$
(5,6)	$2(5pq-pq^2-2p-2)$	$\frac{5}{10pq+q-p-5},\frac{6}{10pq+q-p-6}$
(6,6)	$q(6q-15-q^2)+p(7q+q^2-8)+10$	$\frac{6}{10pq+q-p-6},\frac{6}{10pq+q-p-6}$

Theorem 2.1: The general first temperature index of $TUC_4C_8[p,q]$ is $T_1^a(TUC_4C_8[p,q])$

$$= 4p\left[\frac{70pq+7q-7p-20}{(10pq+q-p-2)(10pq+q-p-5)}\right]^a +2q\left[\frac{6}{10pq+q-p-3}\right]^a + \{2pq(5-q)-4(2p-1)\}\left[\frac{80pq+8q-8p-30}{(10pq+q-p-3)(10pq+q-p-5)}\right]^a + \{17pq-(16p+3q)+q^2(1-p)+2\}\left[\frac{90pq+9q-9p-36}{(10pq+q-p-3)(10pq+q-p-6)}\right]^a + \{8p-5pq+pq^2\}\left[\frac{10}{10pq+q-p-5}\right]^a +2(5pq-pq^2-2p-2)\left[\frac{110pq+11q-11p-60}{(10pq+q-p-5)(10pq+q-p-6)}\right]^a + \{q(6q-15-q^2)+p(7q+q^2-8)+10\}\left[\frac{12}{10pq+q-p-6}\right]^a .$$

Proof . By

$T_1^a(TUC_4C_8[p,q])= \sum_{uv\in E(TUC_4C_8[p,q])}[T(u)+T(v)]^a$

Thus by using Table 2.1. we deduce $T_1^a(TUC_4C_8[p,q])=$

$$4p\left[\frac{2}{10pq+q-p-2}+\frac{5}{10pq+q-p-5}\right]^a +2q\left[\frac{3}{10pq+q-p-3}+\frac{3}{10pq+q-p-3}\right]^a +\{2pq(5-q)-4(2p-1)\}\left[\frac{3}{10pq+q-p-3}+\frac{5}{10pq+q-p-5}\right]^a + \{17pq-(16p+3q)+q^2(1-p)+2\}\left[\frac{3}{10pq+q-p-3}+\frac{6}{10pq+q-p-6}\right]^a + \{8p-5pq+pq^2\}\left[\frac{5}{10pq+q-p-5}+\frac{5}{10pq+q-p-5}\right]^a + 2(5pq-pq^2-2p-2)\left[\frac{5}{10pq+q-p-5}+\frac{6}{10pq+q-p-6}\right]^a + \{q(6q-15-q^2)+p(7q+q^2-8)+10\}\left[\frac{6}{10pq+q-p-6}+\frac{6}{10pq+q-p-6}\right]^a$$

$$= 4p\left[\frac{70pq+7q-7p-20}{(10pq+q-p-2)(10pq+q-p-5)}\right]^a +2q\left[\frac{6}{10pq+q-p-3}\right]^a + \{2pq(5-q)-4(2p-1)\}\left[\frac{80pq+8q-8p-30}{(10pq+q-p-3)(10pq+q-p-5)}\right]^a + \{17pq-(16p+3q)+q^2(1-p)+2\}\left[\frac{90pq+9q-9p-36}{(10pq+q-p-3)(10pq+q-p-6)}\right]^a + \{8p-5pq+pq^2\}\left[\frac{10}{10pq+q-p-5}\right]^a +2(5pq-pq^2-2p-2)\left[\frac{110pq+11q-11p-60}{(10pq+q-p-5)(10pq+q-p-6)}\right]^a + \{q(6q-15-q^2)+p(7q+q^2-8)+10\}\left[\frac{12}{10pq+q-p-6}\right]^a$$

We can obtain the first hyper temperature index of $TUC_4C_8[p,q]$ when $a=2$

Corollary 2.1.1: The sum connectivity temperature index of $TUC_4C_8[p,q]$ is $ST(TUC_4C_8[p,q])=$

$$4p\sqrt{\frac{(10pq+q-p-2)(10pq+q-p-5)}{70pq+7q-7p-20}} + 2q\sqrt{\frac{10pq+q-p-3}{6}} + \{2pq(5-q)-4(2p-1)\}\sqrt{\frac{(10pq+q-p-3)(10pq+q-p-5)}{80pq+8q-8p-30}} + \{17pq-(16p+3q)+q^2(1-p)+2\}\sqrt{\frac{(10pq+q-p-3)(10pq+q-p-6)}{90pq+9q-9p-36}} + \{8p-5pq+pq^2\}\sqrt{\frac{10pq+q-p-5}{10}} +2(5pq-pq^2-2p-2)\sqrt{\frac{(10pq+q-p-5)(10pq+q-p-6)}{110pq+11q-11p-60}} + \{q(6q-15-q^2)+p(7q+q^2-8)+10\}\sqrt{\frac{10pq+q-p-6}{12}} .$$

Proof: Directly from Theorem 2.1 when $a=-1/2$.

Theorem 2. 3: The general second temperature index of $TUC_4C_8[p,q]$ is $T_2^a(TUC_4C_8[p,q])=$

$$4p\left[\frac{10}{(10pq+q-p-2)(10pq+q-p-5)}\right]^a +2q\left[\frac{3}{10pq+q-p-3}\right]^{2a} + \{2pq(5-q)-4(2p-1)\}\left[\frac{15}{(10pq+q-p-3)(10pq+q-p-5)}\right]^a + \{17pq-(16p+3q)+q^2(1-p)+2\}\left[\frac{18}{(10pq+q-p-3)(10pq+q-p-6)}\right]^a +$$

$\{8p-5pq+pq^2\}\left[\frac{5}{10pq+q-p-5}\right]^{2a}+2(5pq-pq^2-2p-2)\left[\frac{30}{(10pq+q-p-5)(10pq+q-p-6)}\right]^{a}+\{q(6q-15-q^2)+p(7q+q^2-8)+10\}\left[\frac{6}{10pq+q-p-6}\right]^{2a}$.

Proof . By

$T_2^a(TUC_4C_8[p,q])=\sum_{uv\in E(TUC_4C_8[p,q])}[T(u)\ T(v)]^a$

and by using table 2.1, we obtain general second temperature index of $TUC_4C_8[p,q]$.

We can find the second hyper temperature index of $TUC_4C_8[p,q]$ from general second temperature index by take a=2

Corollary 2.3.1. The product connectivity temperature index of $TUC_4C_8[p,q]$ is $PT(TUC_4C_8[p,q])$=

$$4p\sqrt{\frac{(10pq+q-p-2)(10pq+q-p-5)}{10}}+2q\,\frac{10pq+q-p-3}{3}+\{2pq(5-q)-4(2p-1)\}\sqrt{\frac{(10pq+q-p-3)(10pq+q-p-5)}{15}}+\{17pq-(16p+3q)+q^2(1-p)+2\}\sqrt{\frac{(10pq+q-p-3)(10pq+q-p-6)}{18}}+\{8p-5pq+pq^2\}\frac{10pq+q-p-5}{5}+2(5pq-pq^2-2p-2)\sqrt{\frac{(10pq+q-p-5)(10pq+q-p-6)}{30}}+\{q(6q-15-q^2)+p(7q+q^2-8)+10\}\frac{10pq+q-p-6}{6}$$

Also we can find the reciprocal product connectivity temperature index of $TUC_4C_8[p,q]$ from gen- eral second temperature index by take a=1/2.

Theorem 2.5: The arithmetic-geometric temperature index of $TUC_4C_8[p,q]$is $AGT\ (TUC_4C_8[p,q])$ =

$$p\,\frac{(70pq+7q-7p-20)\sqrt{10(10pq+q-p-2)(10pq+q-p-5)}}{5(10pq+q-p-2)(10pq+q-p-5)}+2q+\{2pq(5-q)-4(2p-1)\}\frac{(20pq+2q-2p-7.5)\sqrt{15(10pq+q-p-3)(10pq+q-p-5)}}{7.5(10pq+q-p-3)(10pq+q-p-5)}+\{17pq-(16p+3q)+q^2(1-p)+2\}\frac{3(10pq+q-p-4)\sqrt{2(10pq+q-p-3)(10pq+q-p-6)}}{4(10pq+q-p-3)(10pq+q-p-6)}+8p-5pq+pq^2+2(5pq-pq^2-2p-2)\frac{(110pq+11q-11p-60)\sqrt{30(10pq+q-p-5)(10pq+q-p-6)}}{60(10pq+q-p-5)(10pq+q-p-6)}+\{q(6q-15-q^2)+p(7q+q^2-8)+10\}$$

Proof . By

$AGT(TUC_4C_8[p,q])=\sum_{uv\in E(TUC_4C_8[p,q])}\frac{T(u)+T(v)}{2\sqrt{T(u)T(v)}}$ and Table 2.1, we deduce $AGT(TUC_4C_8[p,q])$=

$$4p\,\frac{\frac{2}{10pq+q-p-2}+\frac{5}{10pq+q-p-5}}{2\sqrt{\frac{2}{10pq+q-p-2}\times\frac{5}{10pq+q-p-5}}}+2q\,\frac{\frac{3}{10pq+q-p-3}+\frac{3}{10pq+q-p-3}}{2\sqrt{\frac{3}{10pq+q-p-3}\times\frac{3}{10pq+q-p-3}}}+\{2pq(5-q)-4(2p-1)\}\frac{\frac{3}{10pq+q-p-3}+\frac{5}{10pq+q-p-5}}{2\sqrt{\frac{3}{10pq+q-p-3}\times\frac{5}{10pq+q-p-5}}}+$$

$$\{17pq-(16p+3q)+q^2(1-p)+2\}\frac{\frac{3}{10pq+q-p-3}+\frac{6}{10pq+q-p-6}}{2\sqrt{\frac{3}{10pq+q-p-3}\times\frac{6}{10pq+q-p-6}}}+\{8p-5pq+pq^2\}\frac{\frac{5}{10pq+q-p-5}+\frac{5}{10pq+q-p-5}}{2\sqrt{\frac{5}{10pq+q-p-5}\times\frac{5}{10pq+q-p-5}}}+2(5pq-pq^2-2p-2)\frac{\frac{5}{10pq+q-p-5}+\frac{6}{10pq+q-p-6}}{2\sqrt{\frac{5}{10pq+q-p-5}\times\frac{6}{10pq+q-p-6}}}+\{q(6q-15-q^2)+p(7q+q^2-8)+10\}\frac{\frac{6}{10pq+q-p-6}+\frac{6}{10pq+q-p-6}}{2\sqrt{\frac{6}{10pq+q-p-6}\times\frac{6}{10pq+q-p-6}}}$$ اكتب المعادلة هنا.

$$=p\,\frac{(70pq+7q-7p-20)\sqrt{10(10pq+q-p-2)(10pq+q-p-5)}}{5(10pq+q-p-2)(10pq+q-p-5)}+2q+\{2pq(5-q)-4(2p-1)\}\frac{(20pq+2q-2p-7.5)\sqrt{15(10pq+q-p-3)(10pq+q-p-5)}}{7.5(10pq+q-p-3)(10pq+q-p-5)}+\{17pq-(16p+3q)+q^2(1-p)+2\}\frac{3(10pq+q-p-4)\sqrt{2(10pq+q-p-3)(10pq+q-p-6)}}{4(10pq+q-p-3)(10pq+q-p-6)}+8p-5pq+pq^2+2(5pq-pq^2-2p-2)\frac{(110pq+11q-11p-60)\sqrt{30(10pq+q-p-5)(10pq+q-p-6)}}{60(10pq+q-p-5)(10pq+q-p-6)}+\{q(6q-15-q^2)+p(7q+q^2-8)+10\}.$$

Theorem 2.7: The general temperature index of $TUC_4C_8[p,q]$ is given by $T_a(TUC_4C_8[p,q])$=

$$4p\left\langle\left(\frac{2}{10pq+q-p-2}\right)^a+\left(\frac{5}{10pq+q-p-5}\right)^a\right\rangle+2q\left\langle 2\left(\frac{3}{10pq+q-p-3}\right)^a\right\rangle+\{2pq(5-q)-4(2p-1)\}\left\langle\left(\frac{3}{10pq+q-p-3}\right)^a+\left(\frac{5}{10pq+q-p-5}\right)^a\right\rangle+\{17pq-(16p+3q)+q^2(1-p)+2\}\left\langle\left(\frac{3}{10pq+q-p-3}\right)^a+\left(\frac{6}{10pq+q-p-6}\right)^a\right\rangle+\{8p-5pq+pq^2\}\left\langle 2\left(\frac{5}{10pq+q-p-5}\right)^a\right\rangle+2(5pq-pq^2-2p-2)\left\langle\left(\frac{5}{10pq+q-p-5}\right)^a+\left(\frac{6}{10pq+q-p-6}\right)^a\right\rangle+\{q(6q-15-q^2)+p(7q+q^2-8)+10\}\left\langle 2\left(\frac{6}{10pq+q-p-6}\right)^a\right\rangle.$$

Proof . By

$T_a(TUC_4C_8[p,q])=\sum_{uv\in E(TUC_4C_8[p,q])}[T(u)^a+T(v)^a]$

and using Table 2.1, we obtain the general temperature index of $TUC_4C_8[p,q]$.

We can find The F-temperature index of $TUC_4C_8[p,q]$from the general temperature index by taking a=2

Theorem 2.9: The geometric- arithmetic temperature index of $TUC_4C_8[p,q]$ is

$GAT\ (TUC_4C_8[p,q])$

$$= 4p\frac{2\sqrt{10(10pq+q-p-2)(10pq+q-p-5)}}{70pq+7q-7p-20} + 2q + \{2pq(5-q) - 4(2p-1)\}\frac{\sqrt{15(10pq+q-p-3)(10pq+q-p-5)}}{40pq+4q-4p-15} + \{17pq-(16p+3q)+q^2(1-p)+2\}\frac{2\sqrt{2(10pq+q-p-3)(10pq+q-p-6)}}{30pq+3q-3p-12} + 8p-5pq+pq^2+2(5pq-pq^2-2p-2)\frac{2\sqrt{30(10pq+q-p-5)(10pq+q-p-6)}}{110pq+11q-11p-60} + q(6q-15-q^2)+p(7q+q^2-8)+10 \; .$$

Proof . By $GAT(TUC_4C_8[p,q])=\sum_{uv\in E(TUC_4C_8[p,q])}\frac{2\sqrt{T(u)T(v)}}{T(u)+T(v)}$ and using Table 2.1 we deduce

$$GAT(TUC_4C_8[p,q]) = 4p\frac{2\sqrt{\left(\frac{2}{10pq+q-p-2}\right)\left(\frac{5}{10pq+q-p-5}\right)}}{\left(\frac{2}{10pq+q-p-2}\right)+\left(\frac{5}{10pq+q-p-5}\right)} + 2q\frac{2\sqrt{\left(\frac{3}{10pq+q-p-3}\right)\left(\frac{3}{10pq+q-p-3}\right)}}{\left(\frac{3}{10pq+q-p-3}\right)+\left(\frac{3}{10pq+q-p-3}\right)} + \{2pq(5-q) - 4(2p-1)\}\frac{2\sqrt{\left(\frac{3}{10pq+q-p-3}\right)\left(\frac{5}{10pq+q-p-5}\right)}}{\left(\frac{3}{10pq+q-p-3}\right)+\left(\frac{5}{10pq+q-p-5}\right)} + \{17pq-(16p+3q)+q^2(1-p)+2\}\frac{2\sqrt{\left(\frac{3}{10pq+q-p-3}\right)\left(\frac{6}{10pq+q-p-6}\right)}}{\left(\frac{3}{10pq+q-p-3}\right)+\left(\frac{6}{10pq+q-p-6}\right)} + \{8p-5pq+pq^2\}\frac{2\sqrt{\left(\frac{5}{10pq+q-p-5}\right)\left(\frac{5}{10pq+q-p-5}\right)}}{\left(\frac{5}{10pq+q-p-5}\right)+\left(\frac{5}{10pq+q-p-5}\right)} + 2(5pq-pq^2-2p-2)\frac{2\sqrt{\left(\frac{5}{10pq+q-p-5}\right)\left(\frac{6}{10pq+q-p-6}\right)}}{\left(\frac{5}{10pq+q-p-5}\right)+\left(\frac{6}{10pq+q-p-6}\right)} + \{q(6q-15-q^2)+p(7q+q^2-8)+10\}\frac{2\sqrt{\left(\frac{6}{10pq+q-p-6}\right)\left(\frac{6}{10pq+q-p-6}\right)}}{\left(\frac{6}{10pq+q-p-6}\right)+\left(\frac{6}{10pq+q-p-6}\right)}$$

$$= 4p\frac{2\sqrt{10(10pq+q-p-2)(10pq+q-p-5)}}{70pq+7q-7p-20} + 2q + \{2pq(5-q) - 4(2p-1)\}\frac{\sqrt{15(10pq+q-p-3)(10pq+q-p-5)}}{40pq+4q-4p-15} + \{17pq-(16p+3q)+q^2(1-p)+2\}\frac{2\sqrt{2(10pq+q-p-3)(10pq+q-p-6)}}{30pq+3q-3p-12} + 8p-5pq+pq^2+2(5pq-pq^2-2p-2)\frac{2\sqrt{30(10pq+q-p-5)(10pq+q-p-6)}}{110pq+11q-11p-60} + q(6q-15-q^2)+p(7q+q^2-8)+10 \; .$$

Theorem 2.11: The atom bond connectivity temperature index of $TUC_4C_8[p,q]$ is $ABCT(TUC_4C_8[p,q])$

$$=4p\sqrt{\frac{70pq+7q-7p-20-2\{(10pq+q-p-2)(10pq+q-p-5)\}}{10}} + 2q\frac{\sqrt{\{6-2(10pq+q-p-3)\}(10pq+q-p-3)}}{3} + \{2pq(5-q) - 4(2p-1)\}\sqrt{\frac{80pq+8q-8p-30-2\{(10pq+q-p-3)(10pq+q-p-5)\}}{15}} + \{17pq-(16p+3q)+q^2(1-p)+2\}\sqrt{\frac{90pq+9q-9p-36-2\{(10pq+q-p-3)(10pq+q-p-6)\}}{18}} + \{8p-5pq+pq^2\}\frac{\sqrt{\{10-2(10pq+q-p-5)\}(10pq+q-p-5)}}{5} + 2(5pq-pq^2-2p-2)\sqrt{\frac{110pq+11q-11p-60-2\{(10pq+q-p-5)(10pq+q-p-6)\}}{30}} + \{q(6q-15-q^2)+p(7q+q^2-8)+10\}\frac{\sqrt{\{12-2(10pq+q-p-6)\}(10pq+q-p-6)}}{6} \; .$$

Proof : By $ABCT(TUC_4C_8[p,q])=\sum_{uv\in E(TUC_4C_8[p,q])}\sqrt{\frac{T(u)+T(v)-2}{T(u)T(v)}}$, and using Table 2.1 we deduce $ABCT(TUC_4C_8[p,q])=$

$$4p\sqrt{\frac{\left(\frac{2}{10pq+q-p-2}\right)+\left(\frac{5}{10pq+q-p-5}\right)-2}{\left(\frac{2}{10pq+q-p-2}\right)\left(\frac{5}{10pq+q-p-5}\right)}} + 2q\sqrt{\frac{\left(\frac{3}{10pq+q-p-3}\right)+\left(\frac{3}{10pq+q-p-3}\right)-2}{\left(\frac{3}{10pq+q-p-3}\right)\left(\frac{3}{10pq+q-p-3}\right)}} + \{2pq(5-q) - 4(2p-1)\}\sqrt{\frac{\left(\frac{3}{10pq+q-p-3}\right)+\left(\frac{5}{10pq+q-p-5}\right)-2}{\left(\frac{3}{10pq+q-p-3}\right)\left(\frac{5}{10pq+q-p-5}\right)}} + \{17pq-(16p+3q)+q^2(1-p)+2\}\sqrt{\frac{\left(\frac{3}{10pq+q-p-3}\right)+\left(\frac{6}{10pq+q-p-6}\right)-2}{\left(\frac{3}{10pq+q-p-3}\right)\left(\frac{6}{10pq+q-p-6}\right)}} + \{8p-5pq+pq^2\}\sqrt{\frac{\left(\frac{5}{10pq+q-p-5}\right)+\left(\frac{5}{10pq+q-p-5}\right)-2}{\left(\frac{5}{10pq+q-p-5}\right)\left(\frac{5}{10pq+q-p-5}\right)}} + 2(5pq-pq^2-2p-2)\sqrt{\frac{\left(\frac{5}{10pq+q-p-5}\right)+\left(\frac{6}{10pq+q-p-6}\right)-2}{\left(\frac{5}{10pq+q-p-5}\right)\left(\frac{6}{10pq+q-p-6}\right)}} + \{q(6q-15-q^2)+p(7q+q^2-8)+10\}\sqrt{\frac{\left(\frac{6}{10pq+q-p-6}\right)+\left(\frac{6}{10pq+q-p-6}\right)-2}{\left(\frac{6}{10pq+q-p-6}\right)\left(\frac{6}{10pq+q-p-6}\right)}}$$

$$= 4p\sqrt{\frac{70pq+7q-7p-20-2\{(10pq+q-p-2)(10pq+q-p-5)\}}{10}} + 2q\frac{\sqrt{\{6-2(10pq+q-p-3)\}(10pq+q-p-3)}}{3} + \{2pq(5-q) - 4(2p-1)\}\sqrt{\frac{80pq+8q-8p-30-2\{(10pq+q-p-3)(10pq+q-p-5)\}}{15}} + \{17pq-(16p+3q)+q^2(1-p)+2\}\sqrt{\frac{90pq+9q-9p-36-2\{(10pq+q-p-3)(10pq+q-p-6)\}}{18}} + \{8p-5pq+pq^2\}\frac{\sqrt{\{10-2(10pq+q-p-5)\}(10pq+q-p-5)}}{5} + 2(5pq-pq^2-2p-2)\sqrt{\frac{110pq+11q-11p-60-2\{(10pq+q-p-5)(10pq+q-p-6)\}}{30}} + \{q(6q-15-q^2)+p(7q+q^2-8)+10\}\frac{\sqrt{\{12-2(10pq+q-p-6)\}(10pq+q-p-6)}}{6} \; .$$

Theorem 2.2 The general multiplicative first temperature index of $TUC_4C_8[p,q]$ is

$$TII_1^a(TUC_4C_8[p,q])= \left[\frac{70pq+7q-7p-20}{(10pq+q-p-2)(10pq+q-p-5)}\right]^{4ap}$$
$$\times\left[\frac{6}{10pq+q-p-3}\right]^{2aq}$$
$$\times\left[\frac{80pq+8q-8p-30}{(10pq+q-p-3)(10pq+q-p-5)}\right]^{a\{2pq(5-q)-4(2p-1)\}}\times$$
$$\left[\frac{90pq+9q-9p-36}{(10pq+q-p-3)(10pq+q-p-6)}\right]^{a\{17pq-(16p+3q)+q^2(1-p)+2\}}$$
$$\times\left[\frac{10}{10pq+q-p-5}\right]^{a\{8p-5pq+pq^2\}}$$
$$\times\left[\frac{110pq+11q-11p-60}{(10pq+q-p-5)(10pq+q-p-6)}\right]^{a\{2(5pq-pq^2-2p-2)\}}\times$$
$$\left[\frac{12}{10pq+q-p-6}\right]^{a\{q(6q-15-q^2)+p(7q+q^2-8)+10\}}.$$

proof : The proof direct from
$TII_1^a(TUC_4C_8[p,q])= \prod_{uv\in E(TUC_4C_8[p,q])}[T(u)+T(v)]^a$
and Theorem 2.1

The multiplicative first temperature and first multiplicative hyper temperature indices are obtain from general multiplicative first temperature index by take a=1,2 respectively .

Corollary 2.2.1: The multiplicative sum connectivity temperature index of $TUC_4C_8[p,q]$ is

$$STII(TUC_4C_8[p,q])= \left[\frac{(10pq+q-p-2)(10pq+q-p-5)}{70pq+7q-7p-20}\right]^{2p}$$
$$\times\left[\frac{10pq+q-p-3}{6}\right]^{q}\times$$
$$\left[\sqrt{\frac{(10pq+q-p-3)(10pq+q-p-5)}{80pq+8q-8p-30}}\right]^{2pq(5-q)-4(2p-1)}\times$$
$$\left[\sqrt{\frac{(10pq+q-p-3)(10pq+q-p-6)}{90pq+9q-9p-36}}\right]^{17pq-(16p+3q)+q^2(1-p)+2}$$
$$\times\left[\sqrt{\frac{10pq+q-p-5}{10}}\right]^{8p-5pq+pq^2}\times$$
$$\left[\sqrt{\frac{(10pq+q-p-5)(10pq+q-p-6)}{110pq+11q-11p-60}}\right]^{2(5pq-pq^2-2p-2)}$$
$$\times\left[\sqrt{\frac{10pq+q-p-6}{12}}\right]^{q(6q-15-q^2)+p(7q+q^2-8)+10}.$$

Proof From Theorem 2.2 , we can prove directly when a=-1/2.

Theorem 2.4 The general multiplicative second temperature index of $TUC_4C_8[p,q]$ is

$$TII_2^a(TUC_4C_8[p,q])= \left[\frac{10}{(10pq+q-p-2)(10pq+q-p-5)}\right]^{4ap}$$
$$\times\left[\frac{3}{10pq+q-p-3}\right]^{4aq}\times$$
$$\left[\frac{15}{(10pq+q-p-3)(10pq+q-p-5)}\right]^{a\{2pq(5-q)-4(2p-1)\}}\times$$
$$\left[\frac{18}{(10pq+q-p-3)(10pq+q-p-6)}\right]^{a\{17pq-(16p+3q)+q^2(1-p)+2\}}$$
$$\times$$
$$\left[\frac{5}{10pq+q-p-5}\right]^{2a\{8p-5pq+pq^2\}}$$
$$\times\left[\frac{30}{(10pq+q-p-5)(10pq+q-p-6)}\right]^{2a(5pq-pq^2-2p-2)}\times$$
$$\left[\frac{6}{10pq+q-p-6}\right]^{2a\{q(6q-15-q^2)+p(7q+q^2-8)+10\}}.$$

Proof : From $TII_2^a(TUC_4C_8[p,q])$ $= \prod_{uv\in E(TUC_4C_8[p,q])}[T(u)\ T(v)]^a$,and Theorem 2.3 ,we have general multiplicative second temperature index of $TUC_4C_8[p,q]$

Also , the multiplicative second temperature, multiplicative second hyper temperature, multiplicative product connectivity temperature and reciprocal multiplicative product connectivity temperature index by take a=1,2,-1/2,1/2

Theorem 2.6: The multiplicative arithmetic-geometric temperature index of $TUC_4C_8[p,q]$ is $AGTII(TUC_4C_8[p,q])$

$$= \left[\frac{(70pq+7q-7p-20)\sqrt{10(10pq+q-p-2)(10pq+q-p-5)}}{5(10pq+q-p-2)(10pq+q-p-5)}\right]^{p}\times$$
$$\left[\frac{(20pq+2q-2p-7.5)\sqrt{15(10pq+q-p-3)(10pq+q-p-5)}}{7.5(10pq+q-p-3)(10pq+q-p-5)}\right]^{2pq(5-q)-4(2p-1)}\times$$
$$\left[\frac{3(10pq+q-p-4)\sqrt{2(10pq+q-p-3)(10pq+q-p-6)}}{4(10pq+q-p-3)(10pq+q-p-6)}\right]^{17pq-(16p+3q)+q^2(1-p)+2}$$
$$\times\left[\frac{(110pq+11q-11p-60)\sqrt{30(10pq+q-p-5)(10pq+q-p-6)}}{60(10pq+q-p-5)(10pq+q-p-6)}\right]^{2(5pq-pq^2-2p-2)}$$
.

Proof : From
$AGTII(TUC_4C_8[p,q])=\prod_{uv\in E(TUC_4C_8[p,q])}\frac{T(u)+T(v)}{2\ \sqrt{T(u)T(v)}}$ and Theorem 2.5 we find the multiplicative arithmetic-geometric temperature index of $TUC_4C_8[p,q]$

Theorem 2.8: The general multiplicative temperature index of $TUC_4C_8[p,q]$ is given by

$$TII_a(TUC_4C_8[p,q]) = \left[\left(\frac{2}{10pq+q-p-2}\right)^a + \left(\frac{5}{10pq+q-p-5}\right)^a\right]^{4p}\times\left[2\left(\frac{3}{10pq+q-p-3}\right)^a\right]^{2q}\times$$
$$\left[\left(\frac{3}{10pq+q-p-3}\right)^a + \left(\frac{5}{10pq+q-p-5}\right)^a\right]^{2pq(5-q)-4(2p-1)}$$
$$\times$$
$$\left[\left(\frac{3}{10pq+q-p-3}\right)^a + \left(\frac{6}{10pq+q-p-6}\right)^a\right]^{17pq-(16p+3q)+q^2(1-p)+2}\times$$
$$\left[2\left(\frac{5}{10pq+q-p-5}\right)^a\right]^{8p-5pq+pq^2}\times\left[\left(\frac{5}{10pq+q-p-5}\right)^a + \left(\frac{6}{10pq+q-p-6}\right)^a\right]^{2(5pq-pq^2-2p-2)}$$
$$\left[2\left(\frac{6}{10pq+q-p-6}\right)^a\right]^{q(6q-15-q^2)+p(7q+q^2-8)+10}.$$

Proof . From
$TII_a(TUC_4C_8[p,q])= \prod_{uv\in E(TUC_4C_8[p,q])}[T(u)^a + T(v)^a]$,and Theorem 2.7 we obtain the general multiplicative temperature index of $TUC_4C_8[p,q]$.

We can obtaind the multiplicative F-temperature index of $TUC_4C_8[p,q]$ from Theorem 2.8 by take a=2.

Theorem 2.10: The multiplicative geometric-arithmetic temperature index of $TUC_4C_8[p,q]$ is $GATII(TUC_4C_8[p,q])$

$$= \left[\frac{2\ \sqrt{10(10pq+q-p-2)(10pq+q-p-5)}}{70pq+7q-7p-20}\right]^{4p}\times$$
$$\left[\frac{\sqrt{15(10pq+q-p-3)(10pq+q-p-5)}}{40pq+4q-4p-15}\right]^{2pq(5-q)-4(2p-1)}\times$$
$$\left[\frac{2\ \sqrt{2(10pq+q-p-3)(10pq+q-p-6)}}{30pq+3q-3p-12}\right]^{17pq-(16p+3q)+q^2(1-p)+2}$$
$$\times$$

$$\left[\frac{2\sqrt{30(10pq+q-p-5)(10pq+q-p-6)}}{110pq+11q-11p-60}\right]^{2(5pq-pq^2-2p-2)}.$$

Proof . By

$GATII(TUC_4C_8[p,q])=\prod_{uv\in E(TUC_4C_8[p,q])}\frac{2\sqrt{T(u)T(v)}}{T(u)+T(v)}$, and

Table 2.1 we deduce

$$GATII(TUC_4C_8[p,q])=\left[\frac{2\sqrt{\left(\frac{2}{10pq+q-p-2}\right)\left(\frac{5}{10pq+q-p-5}\right)}}{\left(\frac{2}{10pq+q-p-2}\right)+\left(\frac{5}{10pq+q-p-5}\right)}\right]^{4p}$$

$$\times\left[\frac{2\sqrt{\left(\frac{3}{10pq+q-p-3}\right)\left(\frac{3}{10pq+q-p-3}\right)}}{\left(\frac{3}{10pq+q-p-3}\right)+\left(\frac{3}{10pq+q-p-3}\right)}\right]^{2q}\times$$

$$\left[\frac{2\sqrt{\left(\frac{3}{10pq+q-p-3}\right)\left(\frac{5}{10pq+q-p-5}\right)}}{\left(\frac{3}{10pq+q-p-3}\right)+\left(\frac{5}{10pq+q-p-5}\right)}\right]^{2pq(5-q)-4(2p-1)}\times$$

$$\left[\frac{2\sqrt{\left(\frac{3}{10pq+q-p-3}\right)\left(\frac{6}{10pq+q-p-6}\right)}}{\left(\frac{3}{10pq+q-p-3}\right)+\left(\frac{6}{10pq+q-p-6}\right)}\right]^{17pq-(16p+3q)+q^2(1-p)+2}\times$$

$$\left[\frac{2\sqrt{\left(\frac{5}{10pq+q-p-5}\right)\left(\frac{5}{10pq+q-p-5}\right)}}{\left(\frac{5}{10pq+q-p-5}\right)+\left(\frac{5}{10pq+q-p-5}\right)}\right]^{8p-5pq+pq^2}\times$$

$$\left[\frac{2\sqrt{\left(\frac{5}{10pq+q-p-5}\right)\left(\frac{6}{10pq+q-p-6}\right)}}{\left(\frac{5}{10pq+q-p-5}\right)+\left(\frac{6}{10pq+q-p-6}\right)}\right]^{2(5pq-pq^2-2p-2)}\times$$

$$\left[\frac{2\sqrt{\left(\frac{6}{10pq+q-p-6}\right)\left(\frac{6}{10pq+q-p-6}\right)}}{\left(\frac{6}{10pq+q-p-6}\right)+\left(\frac{6}{10pq+q-p-6}\right)}\right]^{q(6q-15-q^2)+p(7q+q^2-8)+10}$$

$$=\left[\frac{2\sqrt{10(10pq+q-p-2)(10pq+q-p-5)}}{70pq+7q-7p-20}\right]^{4p}$$

$$\times\left[\frac{\sqrt{15(10pq+q-p-3)(10pq+q-p-5)}}{40pq+4q-4p-15}\right]^{2pq(5-q)-4(2p-1)}\times$$

$$\left[\frac{2\sqrt{2(10pq+q-p-3)(10pq+q-p-6)}}{30pq+3q-3p-12}\right]^{17pq-(16p+3q)+q^2(1-p)+2}$$

$\times$

$$\left[\frac{2\sqrt{30(10pq+q-p-5)(10pq+q-p-6)}}{110pq+11q-11p-60}\right]^{2(5pq-pq^2-2p-2)}$$

Theorem 2.12: The multiplicative atom bond connectivity temperature index of $TUC_4C_8[p,q]$ is $ABCTII(TUC_4C_8[p,q])$

$$=\left[\sqrt{\frac{70pq+7q-7p-20-2\{(10pq+q-p-2)(10pq+q-p-5)\}}{10}}\right]^{4p}\times$$

$$\left[\frac{\sqrt{\{6-2(10pq+q-p-3)\}(10pq+q-p-3)}}{3}\right]^{2q}\times$$

$$\left[\sqrt{\frac{80pq+8q-8p-30-2\{(10pq+q-p-3)(10pq+q-p-5)\}}{15}}\right]^{2pq(5-q)-4(2p-1)}\times$$

$$\left[\sqrt{\frac{90pq+9q-9p-36-2\{(10pq+q-p-3)(10pq+q-p-6)\}}{18}}\right]^{17pq-(16p+3q)+q^2(1-p)+2}$$

$\times$

$$\left[\frac{\sqrt{\{10-2(10pq+q-p-5)\}(10pq+q-p-5)}}{5}\right]^{8p-5pq+pq^2}\times$$

$$\left[\sqrt{\frac{110pq+11q-11p-60-2\{(10pq+q-p-5)(10pq+q-p-6)\}}{30}}\right]^{2(5pq-pq^2-2p-2)}\times$$

$$\left[\frac{\sqrt{\{12-2(10pq+q-p-6)\}(10pq+q-p-6)}}{6}\right]^{q(6q-15-q^2)+p(7q+q^2-8)+10}$$

.

Proof: From

$ABCTII(TUC_4C_8[p,q])=$

$\prod_{uv\in E(TUC_4C_8[p,q])}\sqrt{\frac{T(u)+T(v)-2}{T(u)T(v)}}$,and Theorem 2.11,we find

the multiplicative atom bond connectivity temperature index of $TUC_4C_8[p,q]$

3. Conclusion

In this paper we computed the first and second hyper temperature indices, sum connectivity temper- ature index, product connectivity temperature index, arithmetic-geometric temperature index and many of types for temperature indices of $TUC_4C_8[p,q]$ nanotubes .

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مؤشرات درجة الحرارة ودرجة الحرارة المضاعفة للأنابيب النانوية $TUC_4C_8[p,q]$

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الملخص

المؤشرات الطوبولوجية هي أرقام حقيقية تتعلق بالرسم البياني، وتسمى أحيانًا كرسم بياني – مؤشر نظري، هو رقم ثابت من الرسم البياني الكيميائي. وتشمل المؤشرات الطوبولوجية الخاصة أنواع درجات الحرارة ومؤشرات درجة الحرارة المتعددة مثل درجة الحرارة المفرطة الأولى والثانية، ودرجات الحرارة الفائقة المتعددة الأولى والثانية وغيرها. في هذه الورقة نقوم بحساب العديد من أنواع مؤشرات درجة الحرارة ل($TUC_4C_8[p,q]$)