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Application Methods of Linear Feedback Control on the Modified Lorenz 3D Chaotic System

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ABSTRACT

In this paper we using a linear feedback control methods on the Lorenz modified 3D system. we used the dynamical analysis system to find critical points, eigenvalues, and for stability, the Jacobi matrix, to investigate some dynamical behaviors of the system to find stability regions of the system in all methods used to suppress the originally unstable behavior of this system. The necessary and sufficient condition for suppression of unstable behavior for this system is getting positive feedback coefficient. Theoretical analysis and numerical simulation proved that the results obtained.

1. Introduction

A chaotic dynamicalal system was introduced in 1960 by American Edward Lorenz when he was working on weather forecasts. He realized that the sudden results of the weather system might be due to small changes or differences in the initial points. Because the system depending on the initial condition he called it the "butterfly effect". The dynamicalsystem in real life has multiple uses as in the model of population growth [1],[10].

Chaos theory is a branch of mathematical, focusing on the behavior of dynamical system, the other meaning, chaos theory is the study of the irregular [6], unpredictable behavior of deterministic and nonlinear dynamical system [8], [12]. Dynamical system is a behavior system that changes over time, often responding to external stimulation of forcing [9],[11]. When the three properties: boundedness, infinite recurrence and sensitive dependence on the initial conditions are fulfilled in the dynamical system then it called chaotic. Chaos theory has applications in many scientific and engineering fields such as oscillation, chemical reactions, neural networks, nerves, and electrical circuits [1],[2].

Some researchers have discovered how can modify the system Lorenz has found applications in real life. (Zhou Qi et al and Yan) has introduced the modified Lorenz system and discussed stability of the system and dynamical behavior. In secure communications, the modified Lorenz system shows a promising modulation that has potential applications in this field [13]. Tee and Salleh proposed a new modified on Lorenz system finding that led to the discovery of fixed points for the system and dynamicalal analysis by using complementary – cluster energy – barrier criterion(CCEBC).

2. Helping Result and Definition

In this section, some basic concepts and definitions were presented

2.1 Routh- Hurwitz Theorem (R. H. T)

If all root of the polynomial $\lambda^3 + A\lambda^2 + B\lambda + C = 0$ are negative real part, then the followwing condition are satisfiy [1],[4],[5].

1- $A > 0.$

2- $C > 0$

3- $AB - C > 0.$

2.2 Ordinary Feedback Control

In this method we multiply systems variables by unit control and added to unstable a chaotic system provided equivalents to be corresponding to the added variable[15].

2.3 Dislocate Feedback Control

In this method we multiply system's variables by unit control and added to unstable a chaotic system provided equivalents to be contrary to the added variable [7].

2.4 Enhanced Feedback Control

If multiply system variables by more than one of the unit control and added to unstable a chaotic system provided that equivalents the corresponding to the added variable[10],[12].

2.5 Speed Feedback control

As an added feedback in the displacement control method, the system's independent variable is often multiplied by a coefficient [3].

3. System description

3D modified Lorenz system is defined as follows

$$\begin{cases} \dot{x} = a(by - x) \\ \dot{y} = cx - xz \\ \dot{z} = xy - bz \end{cases} \quad \dots .(1)$$

where x, y and z are variable and a, b and c are real parameters. The following figure (Fig.1) described the attractors for this system [13].

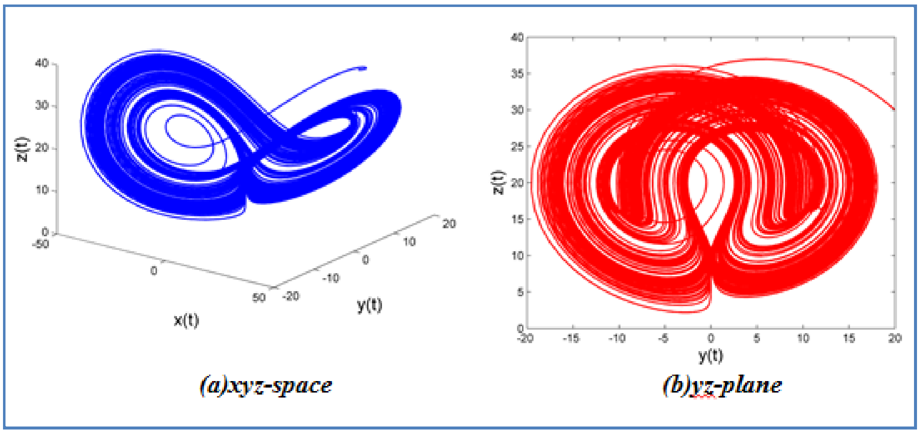

Fig. 1: Attractor of 3D Modified Lorenz system

3.1. System Equilibrium

Assume that $b > 0,\ a^2 + 4abc > 0$, and $\sqrt{a^2 + 4abc} > 0$, The equilibrium of Eq.(1) found by solving three equations $\dot{x} = \dot{y} = \dot{z} = 0$. When a=10, b=4.845 and c=20 the system (1) is chaotic and belongs to the Lorenz system family [13].

$$P_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 21.667 \\ 4.472 \\ 20 \end{bmatrix}, \quad P_3 = \begin{bmatrix} -21.667 \\ -4.472 \\ 20 \end{bmatrix}$$

for system (1) the Jacobian matrix is :

$$J = \begin{bmatrix} -a & ab & 0 \\ c - z & 0 & -x \\ y & x & -b \end{bmatrix} \dots (2)$$

The characteristic equation found by using $\det\,(J - \lambda I) = 0$.

$$|JP_1 - \lambda I| = \begin{vmatrix} -a-\lambda & ab & 0 \\ c & -\lambda & 0 \\ 0 & 0 & -b-\lambda \end{vmatrix} = 0$$

$$\lambda^3 + (a+b)\lambda^2 + (ab - abc)\lambda - ab^2c = 0 \dots .(3)$$

If we substituted P_1 and the value of real constant in the Eq. (3), the eigenvalues are

$\lambda_1 = -b < 0$, $\lambda_2 = \frac{-a-\sqrt{a^2+4abc}}{2} < 0$ and $\lambda_3 = \frac{-a+\sqrt{a^2+4abc}}{2} > 0$. Depending on the result of eigenvalues the behavior of the system in the chaotic system(the system is called chaotic if it has at lest one positive Lyapunov expnent by using Wolf Algorethem) . This means that the system is always unstable with the existence of λ_3 [13].Therefore we can application strategies of linear feedback control (ordinary feedback control, dislocate feedback control, enhanced feedback control and speed feedback control).

3.2. Problem formulation

Assume that the system has the form:

$\dot{X} = AX + h(X) \dots (4)$

Where $X(t) = [x_i]^T = [x_1, x_2, \dots, x_n]^T \in R^{n+1}$, $i = 1,2, \dots n$. $A = (a_{ij})_{n \times n}$ is the matrix parameters and $h : R^n \to R^n$ is the system's nonlinear part. When adding the unit control to the Eq.(3), then the controlled system has the form:

$\dot{X} = AX + h(X) + U \dots .(5)$

The purpose unit control to make $\lim_{t\to\infty} \|X(t)\| = 0$. We can present unit control by the form [8],[9].

$$U = u_i = \begin{cases} -kx_i & ;if\ i = j\ (ordinary) \\ -kx_j & ;if\ i \neq j\ (dislocate) \\ -k\dot{x}_j & ;if\ i \neq j\ \ (speed) \\ -k[x_i]_{i=1}^{n} & ;if\ i = j\ (enhancing) \end{cases}$$

Such that k is feedback parameters and $k > 0$. This strategies depends on Routh Hurwitz Theorem, to find exact value of feedback parameter the necessary and sufficient condition to satisfy chaotic control and the feedback parameter must be positive, some times we get more than positive feedback parameter, to select active feedback parameter we use the equation [10], [15].

$k = \cap_{i=1}^{n} k_i = k_1 \cap k_2 \cap \dots \cap k_n \dots .(6)$

4. Main Result

According to the equation (4), we can written system (1) by the form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a & ab & 0 \\ c & 0 & 0 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ -xz \\ xy \end{bmatrix}$$

this system is chaotic, so, we apply strategies of linear feedback control on above system as the following:

Theorem (1)

The system (1) is converge to unstable equilibrium P_1 when $k \in (96.89, \infty)$ If the control $U = [0 \quad u_2 \quad 0]^T$ where $u_2 = -ky$, and using ordinary feedback control.

Proof : The system(1) and new control can be written in the form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a & ab & 0 \\ c-z & 0 & -x \\ y & x & -b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ -ky \\ 0 \end{bmatrix} \dots (7)$$

Then the characteristic Eq. is:

$$\lambda^3 + A\lambda^2 + B\lambda + C = 0 \dots (8)$$

Where $A = 14.845 + k$, $B = 14.845k - 920.55$ and $C = 48.45k - 4694.8$

By using R. H. T. the Eq. (8) has three eigenvalues all of them are real part and negative. Its clear that, from $A = 14.845 + k > 0$, and $C = 48.45k - 4694.8 > 0$, we conclude $k > 96.89$ as a positive feedback coefficient. Finally, we will get a quadratic equation from the third condition of (R. H. T) as follow:

$$14.845k^2 - 748.626k - 8970.76 > 0 \dots (9)$$

We get another positive feedback coefficient as $k > 60.4295$ from solving the Eq. (9) so that the active and effective feedback control on the system (1) when $k > 96.89$. Fig. 2 show the convergence of this method.

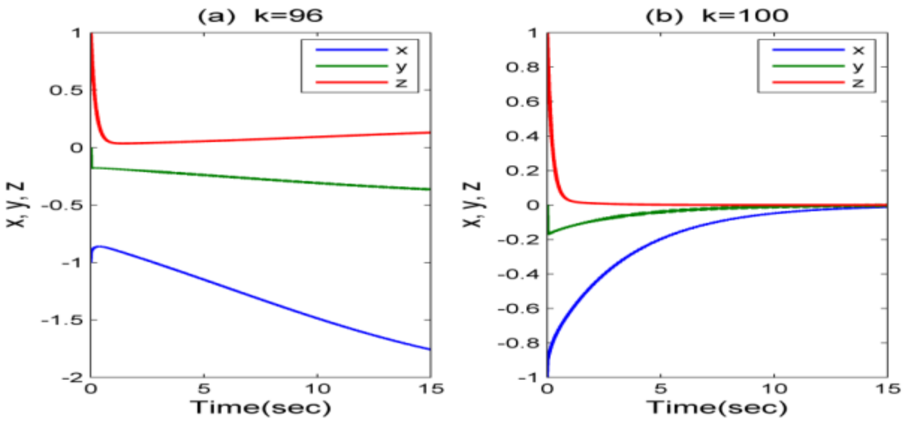

Fig. 2: Convergence of System at P_1

this method fail to control the system (1) when take the control $U = [u_1 \quad 0 \quad 0]^T$ $i.e. U = [-kx \quad 0 \quad 0]^T$ i.e and $U = [0 \quad 0 \quad u_3]^T$ $i.e. U = [0 \quad 0 - kz]^T$ this proof is complete.

Theorem (2)

The system (1) is converge to unstable equilibrium P_1 when $k \in (20, \infty)$ if control $U = [0 \quad u_2 \quad 0]^T$, $u_2 = -kx$ via dislocate feedback control.

Proof: The system(1) and new control can be written in the form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a & ab & 0 \\ c-z & 0 & -x \\ y & x & -b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ -kx \\ 0 \end{bmatrix} \dots (10)$$

Then the characteristic Eq. is:

$$\lambda^3 + 14.845\lambda^2 + (48.45k - 920.55)\lambda + 234.74k - 4694.8 = 0 \dots (11)$$

where $A = 14.845$, $B = 48.45k - 920.55$ and $C = 234.74k - 4694.8$

By using the R. H. T. from $C = 234.74k - 4694.8 > 0$, we conclude $k > 20$ as a positive feedback coefficient. Finally, we get a positive feedback coefficient such that $k > 18.5155$ from the third condition of (R. H. T) , so that the effective and active control on the system (1) is $k > 20$. Fig. 3, show the stability of system (1) at these control depend on above this method.

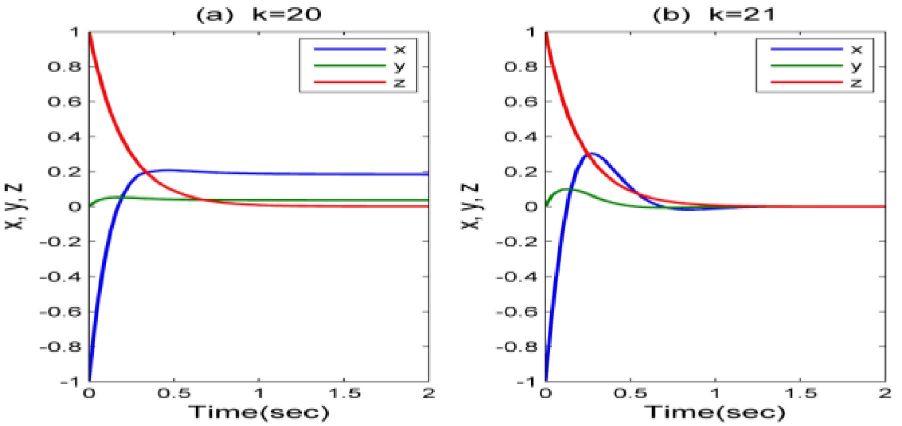

Fig. 3: Convergence of System at P_1

Theorem (3)

system (1) is converge to unstable equilibrium P_1 when $k \in (48.45, \infty)$ if the control is $U = [u_1 \quad 0 \quad 0]^T$ where $u_1 = -ky$, via dislocate feedback control.

Proof : The system(1) and new control can be formed as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a & ab & 0 \\ c-z & 0 & -x \\ y & x & -b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} -ky \\ 0 \\ 0 \end{bmatrix} \quad ....(12)$$

Then the characteristic Eq. is:

$$\lambda^3 + 14.845 \ \lambda^2 + (20k - 920.55)\lambda + 96.9k - 4694.8 \ = 0 \ ....(13)$$

Where $A = 14.845$, $B = 20k - 920.55$ and $C = 96.9k - 4694.8$.

By using the R. H. T. from $C = 96.9k - 4694.8 > 0$, we conclude $k > 48.45$ as a positive feedback coefficient. Finally, we get $k > 44.8538$ as a positive feedback coefficient from the third condition of (R. H. T), so that the effective and active control on the system (1) is $k > 48.45$. Fig. 4, show the stability of system (1) at these control depend on above this method.

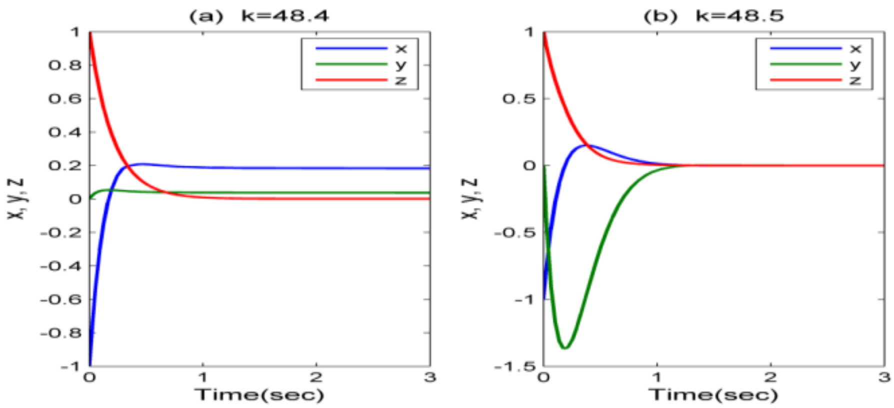

Fig. 4: Convergence of System at P_1

when take another control $U = [0 \quad 0 \quad u_3]^T$, this method fail to control the system (1), this proof is complete.

Theorem (4)

Based on enhancing feedback control, the system (1) is converge to the unstable equilibrium P_1 when $k \in (26.5277, \infty)$. If the control is.

$U = [u_1 \quad u_2 \quad 0]^T$, $u_1 = -kx$, $u_2 = -ky$,

$U = [0 \quad u_2 \quad u_3]^T$, $u_2 = -ky$, $u_3 = -kz$

$U = [u_1 \quad u_2 \quad u_3]^T$, $u_1 = -kx$, $u_2 = -ky$, $u_3 = -kz$

Proof : The system(1) and new control can be written in the form :

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a & ab & 0 \\ c-z & 0 & -x \\ y & x & -b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} -kx \\ -ky \\ 0 \end{bmatrix} \quad ...(14)$$

Then the characteristic Eq. is:

$$\lambda^3 + (14.845 + 2k)\lambda^2 + (k^2 + 19.69k - 920.55)\lambda + 4.845k^2 + 48.45k - 4694.8 = 0 \ ..(15)$$

where $A = 14.845 + 2k$, $B = k^2 + 19.69k - 920.55$ and $C = 4.845k^2 + 48.45k - 4694.8$

By using the R. H. T. from , $C = 234.74k - 4694.8 > 0$, we coclude $k > 26.5277$ as a positive feedback coefficient. Finally, we get $k > 21.6828$ as a positive feedback coefficient from the third condition of (R. H. T), so that the effective and active control on the system (1) is $k > 26.5277$. Fig.5 show the stability of system (1) depend on above control.

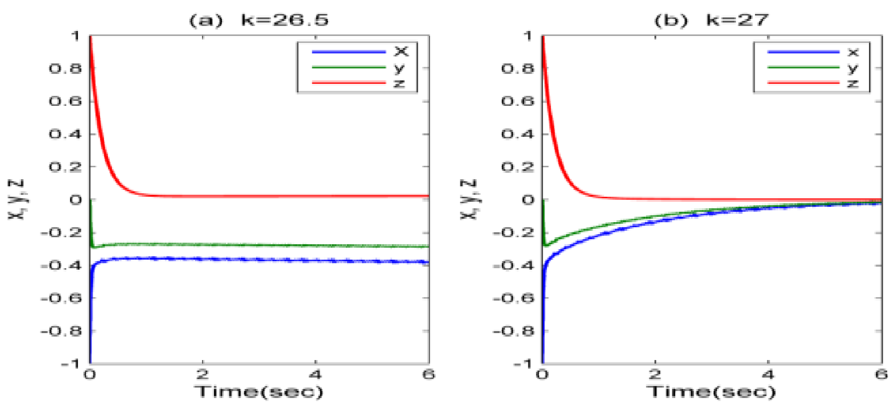

Fig. 5: Stability of System

The alone case fail to control of system (1) and mad him stable when the control is $U = [u_1 \quad 0 \quad u_3]^T$ based on dislocate feedback control, this proof is complete.

Theorem (5)

system (1) is converge to unstable equilibrium P_1 when $k \in (1, \infty)$ if the control is $U = [0 \quad u_2 \quad 0]^T$, $u_2 = -k\dot{x}$ and using speed feedback control.

Proof: The system(1) and new control can be written in the form:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix}-a & ab & 0\\ c-z & 0 & -x\\ y & x & -b\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + \begin{bmatrix}0\\ -k\dot{x}\\ 0\end{bmatrix} \quad \ldots (16)$$

Then the characteristic Eq. is:

$\lambda^3 + (14.845 + 48.45)\ \lambda^2 + (1203.74k - 920.55)\lambda + 4694.8(k-1) = 0 \ \ldots (17)$

where $A = 14.845 + 48.45$, $B = 1203.74k - 920.55$ and $C = 4694.8(k-1)$.

By using the R. H. T. from $A > 0$, $C = 4694.8(k-1) > 0$, we conclude $k > 1$ as a positive feedback coefficient. Finally, we get $k > 0.9257$ as a positive feedback coefficient from the third condition of (R. H. T), so that the effective and active control on the system(1) is $k > 1$, Fig.6 show the stability of system.

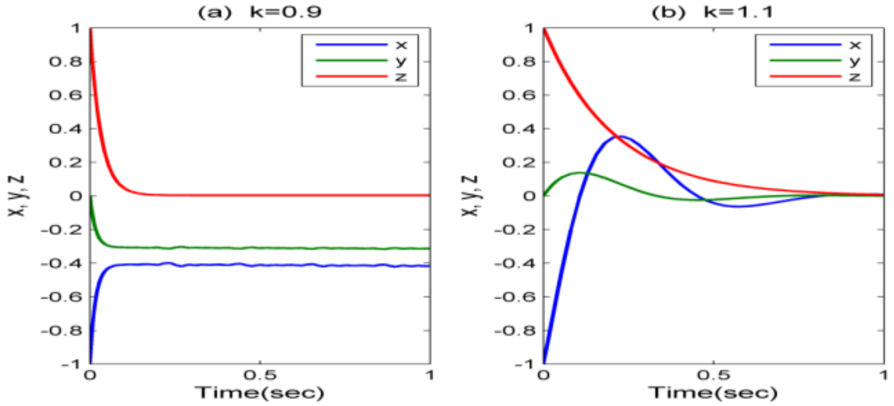

Fig. 6: Stability of system

If the control has the form $U = [u_1 \quad 0 \quad 0]^T$, $u_1 = -k\dot{x}$ although we are achievement conditions of Routh Hurwitz Theorem emergence of one positive feedback coefficient but were unable to get system stable in this case since there was no intersection between the coefficient (if there exist). In this case has been taken one positive feedback coefficient therefore hasn't control on this system [10], also if the control has the form $U = [0 \quad 0 \quad u_3]^T$ these tow cases fail to control the system, the proof is complete.

Conclusion

In this paper we application the methods of linear feedback control on modified Lorenz system at origin point. First we find the critical points of the system and we note that all them is unstable. Second, we applying methods of linear feedback control at origin point. Theoretical analysis show all eigenvalues are negative, so we controlled the behavior of the system in all methods and made him is stable. Finally, numerical simulation result show this modified of Lorenz system can be controlled and mad him stable.

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تطبيق طرق سيطرة التغذية الخطية الرجعية على نظام لورنز المحدث ثلاثي الأبعاد

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الملخص

تم في هذا البحث تطبيق طرق سيطرة التغذية الخطية الرجعية على نظام لورنز المحدث الثلاثي الأبعاد وتم إيجاد النقاط الحرجة باستخدام التحليل الديناميكي للنظام، وإيجاد محدد جاكوبي والقيم الذاتية لاستقرارية النظام للبحث في بعض سلوكيات النظام الديناميكي، وإيجاد مناطق استقرار النظام في جميع الطرق المستخدمة لقمع السلوك الغير المستقر عند نقطة الأصل، حيث أن الشرط الضروري والكافي لقمع السلوك غير المستقر لهذا النظام هو الحصول على عامل تغذية الرجعية موجب وهذا ماتؤكده النتائج النظرية والمحاكاة العددية التي حصلنا عليها.