



A modified three-term conjugate gradient method for large –scale optimization

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ABSTRACT

We propose a three-term conjugate gradient method in this paper . The basic idea is to exploit the good properties of the BFGS update. Quasi – Newton method lies a good efficient numerical computational, so we suggested to be based on BFGS method. However, the descent condition and the global convergent is proven under Wolfe condition. The new algorithm is very effective e for solving the large – scale unconstrained optimization problem.

1. Introduction

In this survey, we concentrate on conjugate gradient (CG) methods for solving the following unconstrained optimization problem

$$\min f(x), x \in \mathbb{R}^n \dots(1.1)$$

Where $f: \mathbb{R}^n \rightarrow \mathbb{R}$ is a continuously differentiable function and its gradient is available. There are many types of numerical methods like the Steepest Descent method, Newton method and Quasi – Newton methods for solving this problem. Conjugate gradient is an efficient method to solve unconstrained optimization problems, especially large-scale problems, as it is characterized by the fact that it does not require large storage and is easy to implement. The conjugated gradient method is implemented numerically as follows:

$$x_{k+1} = x_k + \alpha_k d_k \quad k = 0, 1, \dots(1.2)$$

Where x_k is the approximation solution of (1.1), $\alpha_k > 0$ is a step length and the direction d_k is defined by

$$d_{k+1} = -g_{k+1} + \beta_k d_k, \quad k \geq 1, \dots(1.3)$$

$$d_{k+1} = -g_{k+1}, \quad k = 0, \dots(1.4)$$

Where $g_{k+1} = g(x_k)$ and β_k Is a parameter determined by the CG formula. It is known that the choice of β_k is very important and affects numerical performances of the method.

The most common CG methods are Hestenes - Stiefel (HS)[1], fletcher-reeves (FR)[2], Polak - Ribiere-

Polyak (PRP) [3,4], Dai-Yuan(DY) [5] and Liu and Story (LS) [6]. The method of HS is one of the most effective numerical methods, where his parameter id defined as follows:

$$\beta_k^{HS} = \frac{g_{k+1}^T y_k}{d_k^T y_k} \dots\dots(1.5)$$

Where $y_k = g_{k+1} - g_k$ and $d_{k+1}^T y_k \neq 0$.

Dai and Liao was proposed a new method of HS depended on a different Conjugacy condition in which[7]

$$d_{k+1}^T y_k = -t g_{k+1}^T s_k \dots\dots(1.6)$$

Where $s_k = x_{k+1} - x_k$ or $s_k = \alpha_k d_k$, $t \geq 0$, according to the above condition they are define the following formula of CG

$$\beta_k^{DL} = \frac{g_{k+1}^T y_k}{d_k^T y_k} - t \frac{g_{k+1}^T s_k}{d_k^T y_k}, \quad t \geq 0 \dots\dots(1.7)$$

And they are also update them method according to the truncation technique [8] and with the strong Wolfe line search to get the new CG which is defined as follow

$$\beta_k^{DL+} = \max\left\{\frac{g_{k+1}^T y_k}{d_k^T y_k}, 0\right\} - t \frac{g_{k+1}^T s_k}{d_k^T y_k}, \quad t \geq 0 \dots\dots(1.8)$$

There another suggestion modified of HS was introduced depended on the secant condition predominating contented by Quasi-Newton method. Zhang, Zhou and Li in [9] was proposed as a three term HS in which defined as

$$d_{k+1}^{TTHS} = -g_{k+1} + \beta_k^{HS} d_k - \frac{g_{k+1}^T d_k}{d_k^T y_k} y_k, \quad d_0 = -g_0 \dots (1.9)$$

Li and Fukushima [10] was developed the TTHS method to get a new three term HS method as follows:

$$d_{k+1} = -g_{k+1} + \frac{g_{k+1}^T y_k}{d_k^T z_k} d_k - \frac{g_{k+1}^T d_k}{d_k^T z_k} y_k, \quad d_0 = -g_0 \dots (1.10)$$

$$z_k = y_k + \gamma s_k, \quad \gamma \geq 0 \dots (1.11)$$

Li [11] was defined a new three term HS depended on the memoryless BFGS method [12,13,14] which satisfy $d_{k+1} = -H_k g_{k+1}$

Where

$$H_k = \left(I - \frac{s_k y_k^T}{s_k^T y_k} \right) \left(I - \frac{y_k s_k^T}{s_k^T y_k} \right) + \frac{s_k s_k^T}{s_k^T y_k} \dots (1.12)$$

Where I is the identity matrix

He get the following direction

$$d_{k+1}^{HZDK} = -g_{k+1} + \beta_k^{HZDK} d_k - \lambda_k y_k, \quad d_0 = -g_0 \dots (1.13)$$

$$\beta_k^{HZDK} = \frac{g_{k+1}^T y_k}{d_k^T y_k} - \frac{\|y_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2}, \quad \lambda_k = t_k \frac{g_{k+1}^T y_k}{d_k^T y_k} \dots (1.14)$$

A modified method is presented which satisfies the sufficient descent property independent of the line search and the convexity of the objective function. The global convergence our method is showed for general functions while numerical experiments show the new method is efficient.

2- A New algorithm (ZNZ)

Min Li introduces a new search direction closed to memoryless BFGS method as we mentioned above in section 1, his search direction is as follow

$$\beta_{k+1}^{HZDK} = -g_{k+1} + \beta_k^{HZDK} d_k + \lambda_k y_k, \quad d_0 = -g_0 \dots (1.15)$$

$$\beta_k^{HZDK} = \frac{g_{k+1}^T y_k}{d_k^T y_k} - \frac{\|y_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2} \dots (1.16)$$

$$\lambda_k = t_k \frac{g_{k+1}^T y_k}{d_k^T y_k}, \quad 0 \leq t_k \leq \bar{t} < 1 \dots (1.17)$$

where \bar{t} is a constant to guarantee the sufficient descent property of the new search .In practical computation, we value of

$$t_k = \min\{\bar{t}, \max\{0, 1 - \frac{y_k^T s_k}{\|y_k\|^2}\}\} \dots (1.18)$$

The good qualities of Min Li search direction d_{k+1}^{HZDK} give us a good motivation to find another search direction very close to that direction by use a method similar to the method Dai and kou [7] and Amini and et al [15], we will take the following least – squares problem as follows:

$$\min \|d_{k+1}^{ZNZ} - d_{k+1}^{HZDK}\|^2 \dots (1.19)$$

$$\min \| -g_{k+1} + \beta_k^{ZNZ} d_k - (-g_{k+1} + \beta_k^{HZDK} d_k + \lambda_k y_k) \|^2 \dots (1.20)$$

$$\min \| \beta_k^{ZNZ} d_k - \beta_k^{HZDK} d_k + \lambda_k y_k \|^2 \dots (1.21)$$

$$\min ((\beta_k^{ZNZ})^2 d_k^T d_k - 2\beta_k^{HZDK} \beta_k^{ZNZ} d_k^T d_k + 2\lambda_k \beta_k^{ZNZ} d_k^T y_k + (\beta_k^{HZDK})^2 d_k^T d_k - 2\beta_k^{HZDK} \lambda_k d_k^T y_k + \lambda_k^2 y_k^T y_k) = 0 \dots (1.22)$$

$$2\beta_k^{ZNZ} d_k^T d_k - 2\beta_k^{HZDK} d_k^T d_k + 2\lambda_k d_k^T y_k = 0 \dots (1.23)$$

$$\beta_k^{ZNZ} = \beta_k^{HZDK} - \lambda_k \frac{d_k^T y_k}{d_k^T d_k} \dots (1.24)$$

$$\text{Where } \beta_k^{HZDK} = \frac{g_{k+1}^T y_k}{d_k^T y_k} - \frac{\|y_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2} \dots (1.25)$$

$$\lambda_k = t_k \frac{g_{k+1}^T d_k}{d_k^T y_k}, \quad \leq t_k \leq \bar{t} < 1, \text{ where } t \text{ is constant} \dots (1.26)$$

$$d_{k+1} = -g_{k+1} + \beta_k^{ZNZ} d_k \dots (1.27)$$

Where (1.27) she direction search A new Algorithm.

2.1- Algorithm (The ZNZ Method)

Step 1. Choose an initial value $x_1, \varepsilon > 0$ put $d_1 = -g_k, x_1 = \nabla f_1, k = 1$

Step 2. If it was $\|g_k\| < \varepsilon$ Aim pausing, x_k is the perfect point Otherwise go to step (3)

Step 3. Calculate the length of the step $\alpha_k > 0$ police officer Wolfe

Step 4. Calculate $x_{k+1} = x_k + \alpha_k d_k$ and calculate f_{k+1}, g_{k+1} and $S_k = x_{k+1} - x_k, y_k = g_{k+1} - g_k$.

Step 5. Calculate the search direction d_{k+1} As the equation (1.27) and β by (1.24)

Step 6. Check if the meter $|g_{k+1}^T g_k| > 0.2 \|g_{k+1}\|$ We put $g_{k+1} = -g_{k+1}$.

Step 7. Calculate an initial value $\alpha_{k+1} = \alpha_k \left(\frac{\|d_k\|}{\|d_{k+1}\|} \right)$.

Step 8. Put $k = k + 1$ And go to step (2).

3- The Descent property of the New formal

In the beginning we will mention the property of sufficient regression the new proposed formula conjugate managing algorithm does knew where the coefficient of compatibility in the following format :

$$g_{k+1}^T d_{k+1} \leq -c \|g_{k+1}\|^2 \dots (1.28)$$

Theorem (3.1)

Let d_{k+1} Search direction and $k \geq 0$ it is born with formula (1.27) it is that size of the line and the verification of the condition and the thousand Wolfe then d_{k+1} Check your own beef (1.28).

Proof:

To proof by pattern in the sports induction

1- when $k = 0$ the $d_1 = -g_1 \rightarrow d_1^T g_1 = -\|g_1\|^2 < 0$

2- Assume that relationship (1.28) true all values k .

3- we prove the health of the relationship (1.28) when $k = k + 1$ so that ends of relationship (1.27) d_{k+1} we get

$$d_{k+1} = -g_{k+1} + \beta_k^{ZNZ} d_k \Rightarrow g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \beta_k^{ZNZ} d_k g_{k+1}^T$$

$$\Rightarrow g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \left(\beta_k^{HZDK} - \frac{\lambda_k y_k^T d_k}{\|d_k\|^2} \right) d_k g_{k+1}^T$$

$$\Rightarrow g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \left(\frac{g_{k+1}^T y_k}{d_k^T y_k} - \frac{\|y_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2} + \frac{\lambda_k y_k^T d_k}{d_k^T y_k \|d_k\|^2} \right) d_k g_{k+1}^T$$

$$\Rightarrow g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{(g_{k+1}^T y_k)(g_{k+1}^T d_k)}{d_k^T y_k} - \frac{\|y_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} + t_k \frac{(g_{k+1}^T d_k)^2}{\|d_k\|^2}$$

$$\Rightarrow g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{(g_{k+1}^T y_k)(g_{k+1}^T d_k)}{d_k^T y_k} - \frac{\|y_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} + t_k \frac{(g_{k+1}^T d_k)^2}{\|d_k\|^2}$$

$$\Rightarrow g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{(g_{k+1}^T y_k)(g_{k+1}^T d_k)}{d_k^T y_k} - \frac{\|y_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} + t_k \frac{(g_{k+1}^T d_k)^2}{\|d_k\|^2}$$

$$\Rightarrow g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{(g_{k+1}^T y_k)(g_{k+1}^T d_k)}{d_k^T y_k} - \frac{\|y_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} + t_k \frac{(g_{k+1}^T d_k)^2}{\|d_k\|^2}$$

$$\Rightarrow g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{(g_{k+1}^T y_k)(g_{k+1}^T d_k)}{d_k^T y_k} - \frac{\|y_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} + t_k \frac{(g_{k+1}^T d_k)^2}{\|d_k\|^2}$$

$$\Rightarrow g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{(g_{k+1}^T y_k)(g_{k+1}^T d_k)}{d_k^T y_k} - \frac{\|y_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} + t_k \frac{(g_{k+1}^T d_k)^2}{\|d_k\|^2}$$

By the following property we gat [16].

$$u^T v \leq \frac{1}{2} (\gamma u^2 + \gamma^{-1} v^2), \quad \gamma > 0, \quad u, v \in R^n$$

$$\begin{aligned} &\Rightarrow g_{k+1}^T d_{k+1} = \\ &- \|g_{k+1}\|^2 + 2 \left(\frac{g_{k+1}}{2} \right) \left(y_k \frac{(g_{k+1}^T d_k)}{(d_k^T y_k)} \right) - \\ &\frac{\|y_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} + t_k \frac{(g_{k+1}^T d_k)^2}{\|d_k\|^2} \\ &\Rightarrow g_{k+1}^T d_k \leq -\|g_{k+1}\|^2 + \frac{1}{4} \|g_{k+1}\|^2 + \\ &\frac{\|y_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} - \frac{\|y_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} + \frac{(g_{k+1}^T d_k)^2}{\|d_k\|^2} \end{aligned}$$

By the following property we get

$$\begin{aligned} &\Rightarrow (g_{k+1}^T d_k)^2 \leq \|g_{k+1}\|^2 \|d_k\|^2 \\ &\Rightarrow g_{k+1}^T d_k \leq -\|g_{k+1}\|^2 + \frac{1}{4} \|g_{k+1}\|^2 + \\ &t_k \frac{\|g_{k+1}\|^2 \|d_k\|^2}{\|d_k\|^2} \end{aligned}$$

$$\Rightarrow g_{k+1}^T d_k \leq (-1 + \frac{1}{4} + t_k) \|g_{k+1}\|^2$$

$$c = -\frac{3}{4} + t_k < 0$$

$$\Rightarrow g_{k+1}^T d_{k+1} \leq -c \|g_{k+1}\|^2$$

So the relationship (1.28) Check the condition is adequately.

4- The global Convergent

Assumption (4.1):

(I) The level set $\Omega = \{x \in \mathbb{R}^n : f(x) \leq f(x_0)\}$ is bounded }.

(II) In some neighborhood N of Ω , function f is continuously differentiable and its gradient is Lipschitz continuous, namely, there exists a constant $L > 0$ such that [17].

$$\|g(x) - g(y)\| \leq L \|x - y\|, \forall x, y \in N, \dots(1.29)$$

The assumption implies that are positive constants β and γ_1 such that

$$\|x\| \leq \beta \text{ and } \|g(x)\| \leq \gamma_1, \forall x \in \Omega \dots(1.30)$$

Lemma (4.2):

Suppose that the conditions in Assumption A hold, $\{g_k\}$ and $\{d_k\}$ are generated by TPRP method with the Wolfe line search than [18].

$$\sum_{k=0}^{\infty} \frac{\|g_{k+1}\|^4}{\|d_k\|^2} < +\infty \dots(1.31)$$

Theorem (4.3):

suppose that the conditions in Assumption A hold $\{g_{k+1}\}$ is generated by the TPRP method with the Wolfe line search, then

$$\lim_{k \rightarrow \infty} \|g_{k+1}\| = 0 \dots(1.32)$$

Proof:

$$\begin{aligned} d_{k+1} &= -g_{k+1} + \beta_k^{ZNZ} d_k \\ \Rightarrow \|d_{k+1}\| &\leq \|g_{k+1}\| + \left| \beta_k^{HZDK} + \frac{\lambda_k y_k^T d_k}{d_k^T d_k} \right| \|d_k\| \\ \Rightarrow \|d_{k+1}\| &\leq \|g_{k+1}\| + \left| \frac{g_{k+1}^T y_k}{d_k^T y_k} - \frac{\|y_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2} + \right. \\ &\left. \frac{t_k g_{k+1}^T d_k y_k^T d_k}{\|d_k\|^2 d_k^T y_k} \right| \|d_k\| \\ \Rightarrow \|d_{k+1}\| &\leq \|g_{k+1}\| + \left(\frac{\|g_{k+1}\| \|y_k\|}{d_k^T y_k} - \right. \\ &\left. \frac{\|y_k\|^2 \|g_{k+1}\| \|d_k\|}{(d_k^T y_k)^2} + \frac{t_k \|g_{k+1}\| \|d_k\| \|y_k\| \|d_k\|}{d_k^T y_k \|d_k\|^2} \right) \|d_k\| \\ \Rightarrow \|d_{k+1}\| &\leq \|g_{k+1}\| + (1 + t_k) \frac{\|g_{k+1}\| \|y_k\| \|d_k\|}{d_k^T y_k} - \\ &\frac{\|g_{k+1}\| \|y_k\|^2 \|d_k\|^2}{(d_k^T y_k)^2} \end{aligned}$$

$$\begin{aligned} \Rightarrow \|d_{k+1}\| &\leq \|g_{k+1}\| + \|g_{k+1}\| \left((1 + t_k) \frac{\|d_k\| \|y_k\|}{d_k^T y_k} - \right. \\ &\left. \frac{\|d_k\|^2 \|y_k\|^2}{(d_k^T y_k)^2} \right) \end{aligned}$$

$$\begin{aligned} \Rightarrow \|d_{k+1}\| &\leq \|g_{k+1}\| \left(\left(1 + (1 + t_k) \frac{\|d_k\| \|y_k\|}{d_k^T y_k} \right) - \right. \\ &\left. \frac{\|d_k\|^2 \|y_k\|^2}{(d_k^T y_k)^2} \right) \end{aligned}$$

By the following property we get [16].

$$u^T v \leq \frac{1}{2} (\gamma u^2 + \gamma^{-1} v^2), \gamma > 0 \text{ and } u, v \in \mathbb{R}^n$$

$$\begin{aligned} \Rightarrow \|d_{k+1}\| &\leq \|g_{k+1}\|^2 (1 + (1 + t_k))^2 + \\ &\frac{\|d_k\|^2 \|y_k\|^2}{(d_k^T y_k)^2} - \frac{\|d_k\|^2 \|y_k\|^2}{(d_k^T y_k)^2} \end{aligned}$$

$$\Rightarrow \|d_{k+1}\| \leq c \|g_{k+1}\|^2$$

$$\Rightarrow C = (1 + (1 + t_k))^2$$

$$\sum_{k=0}^{\infty} \|g_{k+1}\|^2 \leq \sum_{k=1}^{\infty} \frac{c^2 \|g_{k+1}\|^4}{\|d_{k+1}\|^2} < \infty$$

$$\lim_{k \rightarrow \infty} \|g_{k+1}\| = 0.$$

Combining this with (1.32) gives

This implies $\lim_{k \rightarrow \infty} \|g_{k+1}\| = 0$. The proof is completed.

5. Numerical experiment

The numerical results of the suggested method ZNZ proved its validity and achieved the condition of Wolf and line search. For a set of test functions in unrestricted optimization.[19]

And to evaluate the performance of this proposed new algorithm that was compared with HZDK by testing 75 function.

The drug was chosen to hold back $n = 100, \dots, 1000$ by comparing the performance of this new proposed algorithm with HZDK, the measure used to stop the repetition of these algorithms is $\|g_k\|^2 = 10^{-6}$. All codes are written with Fortran 77 on PC. The colors are in shapes (1),(2),(3), red and blue in blue represents the proposed new signifier ZNZ and the color is red algorithm HZDK, the test function begin to repeat with the standard starting point and summarize the numerical results recorded in the figures (1),(2),(3), and through the program core(TM) is - 7500 cup, the scale of the algorithm evaluation will be compared based on (Dolan and more) [20].

To compare the efficiency of this proposed algorithm with HZDK. he knew $p = 7500$ a set of n_p test function and $s = 3$ number of algorithm used. let $l_{p,s}$ represents the number of times the value of the target function is found from the algorithm S to solve the problem P.

$$r_{p,s} = \frac{l_{p,s}}{l_p^*} \dots\dots (1.33)$$

Ears $l_p^* = \min\{l_{p,s} : s \in S\}$ it is clear that $r_{p,s} > 1$ per values p, s . if the algorithm fails to solve problems, the ratio $r_{p,s}$ is equal to the large number M, efficiency property of the algorithm s introduces to the accumulated distribution of efficiency ratio

$$p_s(t) = \frac{\text{size}\{p \in P : r_{p,s} \leq t\}}{n_p} \dots\dots(1.34)$$

It is clear $p_s(1)$, represents the percentage of successful algorithm s, efficiency characteristic can

also be used in frequency analysis ,the number , value ,gradient, and processor time . As well as get the situation notes in the following chart .

Note the shape (1),(2),(3), the curve of the algorithm ZNZ is at the top meaning that it needs the fewest number of iterations followed by HZDK , As for the

number of function calculations note that the curve the algorithm ZNZ is at the bottom and this means that it needs the least Numer of function calculations of the HZDK method, As well as there is a clear superiority in terms of time spent in the calculations shown in (3).

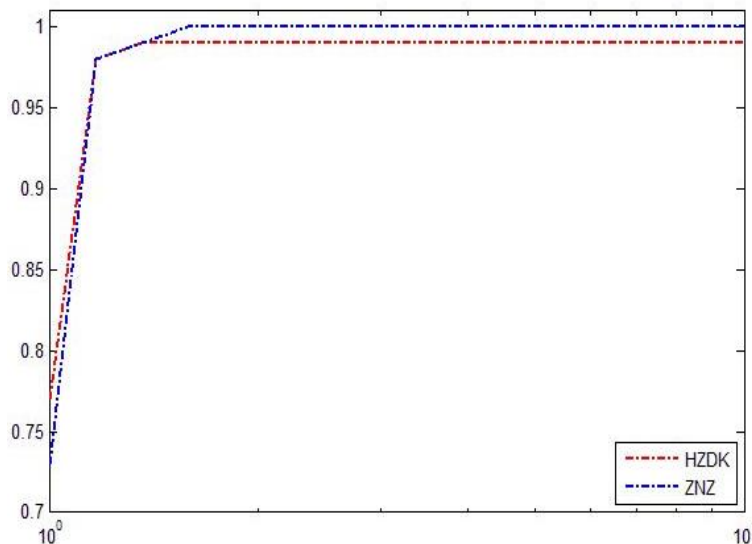


Fig. 1: Performance profiles of iterations

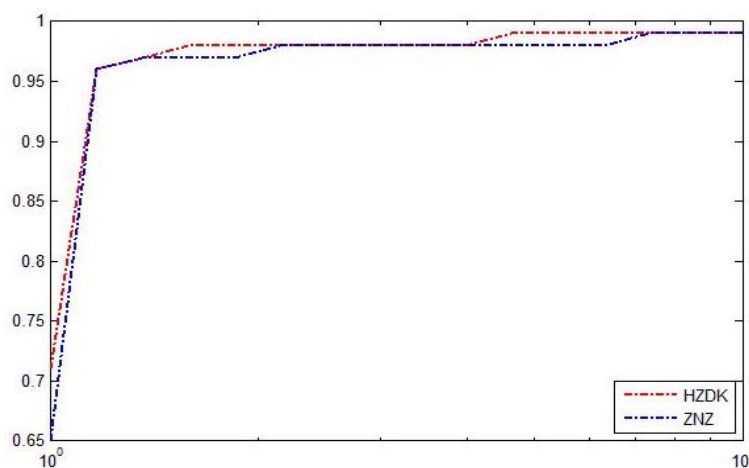


Fig. 2: Performance profiles of function evaluations

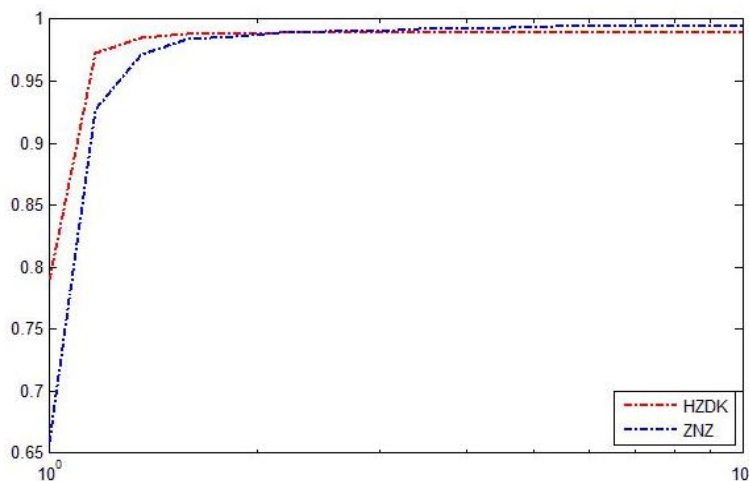


Fig. 3: Performance profiles of cpu time

6. Conclusion

The new function of the new search engine, being Wolfe, showed better results and that is of a three term conjugate gradient method with good property and universal outcome as opposed to the other

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algorithm . The optimal value of β_k^{ZNZ} in the search direction (3) ,(4) may be a good further works of this paper.

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طريقة جديدة للتدرج المترافق الاخطية معتمدة على طريقة BFGS

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الملخص

طريقة التدرج المترافق المعتمدة على طريقة شبيهة نيوتن (BFGS) تعطي نتائج عددية كفوءة ، لذلك اقترحنا طريقة جديدة في هذا البحث معتمدة على طريقة (BFGS) حيث تم اثبات خاصية الانحدار الكافي والتقارب المطلق للطريقة المقترحة تحت شروط ولف . الطريقة الجديدة كفوءة جدا في حل مسائل الأمثلية غير المقيدة ذات القياس العالي .