IASJ
المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science
ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j

New Hybrid Conjugate Gradient Method as a Convex Combination of Dai–Liao and Wei–Yao–Liu

Zeyad Mohammed Abdullah , Faris Saleem Dhyab
Faculty of Computer Science and Mathematics, Department of Mathematics, University of Tikrit, Tikrit, Iraq
https://doi.org/10.25130/tjps.v26i6.199

ARTICLE INFO.
Article history:
-Received: 27 / 5 / 2021
-Accepted: 1/ 9 / 2021
-Available online: / / 2021

Keywords: Unconstrained optimization, Conjugate Gradient method, Inexact line search, Global convergence, Hybrid conjugate gradient method .

Corresponding Author:
Name: **Zeyad Mohammed Abdullah**
E-mail:
zeyaemoh1978@tu.edu.iq
faris.s.dheyab35518@st.tu.edu.iq
Tel:

ABSTRACT

In this research, a new method of hybrid conjugated gradient methods was developed. This method is based mainly on the hybridization of Dia-Laio and Wei-Yao-Liu algorithms, by using convex fitting and conjugate condition of line Uncontrolled search. The resulting algorithm fulfills the condition of sufficient proportions and has universal convergence under certain assumptions. The numerical results indicated the efficiency of this method in solving nonlinear test functions in the given unconstrained optimization.

1. Introduction

Optimization problems are the process of obtaining minimization or maximization of a function from variables Consider the following an unconstrained minimization problem in the form as follows:

$\min_{x \epsilon R^n} f(x)$ (1)

Where $f: R^n \to R$ is continuously differentiable function and bounded below. Nonlinear conjugate gradient method is well suited to solving problems its iterative formula is given by

$x_{k+1} = x_k + \alpha_k d_k \quad k = 0,1,2 \ldots$.(2)

Where the positive step size $\alpha_k > 0$ is obtained by α line search, The search direction d_k on this gradient conjugate method uses the following rules:

$$d_{k+1} = \begin{cases} -g_{k+1} \quad , \quad k = 0 \\ -g_{k+1} + \beta^{FZ}{}_k d_k \end{cases} \quad \ldots (3)$$

where $g_k = \nabla f(x_k)$, and β_k is an important parameter. The different choices for the parameter β_k correspond to different CG methods. Over the years, many variants of this scheme are proposed, and some are widely used in practice.

Above step length α It was obtained with the terms of the SWP (The strong Wolfe–Powell) consist line research as follows :

$|g(x_k + \alpha_k d_k)^T| \leq -\sigma g^T{}_k \, d_k \quad \ldots (4)$

We are now reviewing some popular formulas β_k :

$\beta^{HS}{}_k = \frac{g^T{}_{k+1} y_k}{d^T{}_k \, y_k}$ (Hestenses–Stiefel , 1952 [11]) … (5)

$\beta^{FR}{}_k = \frac{\|g_{k+1}\|^2}{\|g_k\|^2}$ (Fletcher – Reeves , 1964 [12]) ….(6)

$\beta^{PR}{}_k = \frac{g^T{}_{k+1} y_k}{\|g_k\|^2}$ (Polak – Ribiere , 1969 [1]) …(7)

$\beta^{DX}{}_k = \frac{\|g_{k+1}\|^2}{d^T{}_k \, g_k}$ (Dixon , 1975 [4]) …(8)

$\beta^{CD}{}_k = \frac{-\|g_{k+1}\|^2}{g^T{}_k \, d_k}$ (Fletcher- Cd- 1987 [13]) …(9)

$\beta^{LS}{}_k = \frac{g^T{}_{k+1} y_k}{-d^T{}_k \, g_k}$ (Liu-Storey , 1991 [15]) …(10)

$\beta^{DY}{}_k = \frac{\|g_{k+1}\|^2}{y_k{}^T g_k}$ (Dai-Yuan , 1999 [14]) …(11)

The above-mentioned methods are identical when the quadratic function is tightly convex and the line search is accurate, because the gradients are mutually orthogonal, the HS, PR, LS algorithms are effective and efficient in scientific applications. Mathematically, their convergence did not appear as for the other part of the algorithm (6), (8), (9), (11) as it has been theoretically proven that it possesses the property of comprehensive convergence but is inefficient in scientific applications, and through the above preferences a new type of conjugate algorithms called hybrids appeared, under the conditions of SWP, several hybrid methods were proposed In order to obtain conjugate gradient methods which have good computational efficiency and affinity properties. Scientists have found these techniques in order to eliminate failure and improve performance of popular conjugate gradient algorithms. The hybrid CG algorithms are better than the classical conjugated gradient algorithms where the hybridization method is based on the concept of the convex structure of the classical CG algorithms. The θ_k parameter is found in the convex structure by making use of the fulfillment of the conjugation condition and the direction of Newton.[18] . Significant progress has recently been made in the field of conjugate gradient methods and their applications [8 and 4]. We mention the mixed pairing method, which is a specific set of different pairing methods. Designed to improve the behavior of these methods and avoid the phenomenon of perturbation, here we consider a convex combination of the coupled gradient methods DL and WYL, the corresponding paired gradient parameters are

Our study in this paper will be a Hybrid Conjugate Gradient Method as a Convex Combination of Dai–Liao (DL)

and Wei–Yao–Liu (WYL).

$$\beta^{WYL}{}_k = \frac{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2} \quad \dots (12)$$

$$\beta^{DL}{}_k = \frac{(y_k - t s_k)^T g_{k+1}}{d^T{}_k\, y_k}, \; y_k = (g_{k+1} - g_k), \; t = \frac{2\|y_k\|^2}{s^T{}_k y_k} \quad \dots (13)$$

where t is a nonnegative parameter and note that if t=0 then $\beta^{DL}{}_k$ reduces to the CG parameter proposed by (5) Hestenes and Stiefel (1952).

Hybrid by using a convex structure .i.e : $\beta^{FZ}{}_k = \theta_k\beta^{WYL}{}_k + (1-\theta_k)\beta^{DL}{}_k$ …(14)

and by use of the conjugation condition for an uncontrolled search line (Dai and Liao,2001 [2])

that takes the formula θ_k to calculate the parameter :

$d^T{}_{k+1}y_k = -\rho g^T{}_{k+1}S_k \quad , \rho > 0$ …(15)

Now, we introduce a new hybrid method

2- A New Hybrid Conjugate Gradient Method

Hybrid methods are done by connecting two methods, one of the two methods has good computational properties and the second method has strong comprehensive convergence properties.

Using (14), we get the value for the parameter θ_k, which we denote

$$\beta^{FZ}{}_k = \left(\theta_k\beta^{WYL}{}_k + (1-\theta_k)\beta^{DL}{}_k\right)$$

from (12) and (13) We get:

$$\beta^{FZ}{}_k = \theta_k \frac{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2} + (1-\theta_k)\frac{(y_k - t s_k)^T g_{k+1}}{d^T{}_k\, y_k}$$

$$= \theta_k \frac{g^T{}_k\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2} + (1-\theta_k)\frac{g^T{}_{k+1}\left(y_k - \frac{2\|y_k\|^2}{s^T{}_k y_k}s_k\right)}{d^T{}_k\, y_k}$$

$$= \theta_k \frac{g^T{}_k\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2} + (1-\theta_k)\frac{g^T{}_{k+1}\left(y_k - \frac{2y^T{}_k s_k}{s^T{}_k y_k}y_k\right)}{d^T{}_k\, y_k}$$

$$= \theta_k\left(\frac{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2} - \frac{-g^T{}_{k+1}y_k}{d^T{}_k\, y_k}\right) + \frac{-g^T{}_{k+1}y_k}{d^T{}_k\, y_k}$$

From (3) we get :

$$d_{k+1} = -g_{k+1} + \left(\theta_k\left(\frac{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2} - \frac{-g^T{}_{k+1}y_k}{d^T{}_k\, y_k}\right) + \frac{-g^T{}_{k+1}y_k}{d^T{}_k\, y_k}\right)d_k$$

Multiplying both sides of the equation by the vector y_k we get :

$$d^T{}_{k+1}y_k = -g^T{}_{k+1}y_k + \left(\theta_k\left(\frac{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2} + \frac{g^T{}_{k+1}y_k}{d^T{}_k\, y_k}\right) - \frac{g^T{}_{k+1}y_k}{d^T{}_k\, y_k}\right)d^T{}_k y_k$$

Now we substitute the conjugation condition for the uncontrolled search line (15)

.i.e.. $d^T{}_{k+1}y_k = -\rho g^T{}_{k+1}S_k$

$$-\rho g^T{}_{k+1}S_k = -g^T{}_{k+1}y_k + \left(\theta_k\left(\frac{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2} + \frac{g^T{}_{k+1}y_k}{d^T{}_k\, y_k}\right) - \frac{g^T{}_{k+1}y_k}{d^T{}_k\, y_k}\right)d^T{}_k y_k$$

$$-\rho g^T{}_{k+1}S_k + g^T{}_{k+1}y_k = \theta_k\left(\frac{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2}d^T{}_k y_k + g^T{}_{k+1}y_k\right) - g^T{}_{k+1}y_k$$

$$\theta_k = \frac{-\rho g^T{}_{k+1}S_k + 2g^T{}_{k+1}y_k}{\frac{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2}d^T{}_k y_k + g^T{}_{k+1}y_k} = \frac{\|g_{k-1}\|^2\left(-\rho g^T{}_{k+1}S_k + 2g^T{}_{k+1}y_k\right)}{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)d^T{}_k y_k + \|g_{k-1}\|^2 g^T{}_{k+1}y_k}$$

Algorithm $\beta^{FZ}{}_k$

Step 1 : set $x_\circ \in R^n$, $\varepsilon \geq 0$. compute $f(x_\circ)$ and g_0 set $d_0 = -g_0$, and the initial guess $\alpha_0 = \frac{1}{\|g_0\|}$ if $\|g_0\| \leq \varepsilon$, then stop .

Step 2 : Compute α_k by line searches defined in equation (4).

Step 3 : Let $x_{k+1} = x_k + \alpha_k d_k$, $g_{k+1} = g(x_{k+1})$. Compute s_k and y_k

Step 4 : If $0 < \theta_k < 1$ then compute $\beta^{FZ}{}_k$ by (3) if $\theta_k \geq 1$, then compute $\beta^{FZ}{}_k$ by $\beta^{WYL}{}_k$. If $\theta_k \leq 0$ then compute $\beta^{FZ}{}_k$ by $\beta^{DL}{}_k$

Step 5 : Increase k by one and go to Step 2 .

3- Invistigation Sufficient descent

Theorem 1: If the objective function $f(x)$ is continuously differentiable and Let the sequences $\{g_k\}$ and $\{x_k\}$be generated by $\beta^{FZ}{}_k$method. Then the search direction d_k satisfies the sufficient descent condition:

$d^T{}_k g_k \leq -C\|g_k\|^2$ (16)

Where C is a positive constant independent of k,

Proof : The result can be establish by induction. When k=1 we have $d^T{}_1 g_1 \leq -g^T{}_1 g_1 \leq -C\|g_k\|^2$

Let k=k+1

From (3) $d_{k+1} = -g_{k+1} + \beta^{FZ}{}_k d_k$

Multiplying both sides of the equation by the vector g_{k+1} and from (14) we get :

$$d^T{}_{k+1} g_{k+1} = -\|g_{k+1}\|^2 + \left(\theta_k \beta^{WYL}{}_k + (1-\theta_k)\beta^{DL}{}_k\right) d^T{}_k g_{k+1}$$

Let $-\|g_{k+1}\|^2 = -\|g_{k+1}\|^2\theta_k - (1-\theta_k)\|g_{k+1}\|^2$

$$d^T{}_{k+1} g_{k+1} = -\|g_{k+1}\|^2\theta_k - (1-\theta_k)\|g_{k+1}\|^2 + \left(\theta_k \beta^{WYL}{}_k + (1-\theta_k)\beta^{DL}{}_k\right) d^T{}_k g_{k+1}$$

$$d^T{}_{k+1} g_{k+1} = -\|g_{k+1}\|^2\theta_k + \theta_k \beta^{WYL}{}_k d^T{}_k g_{k+1} - (1-\theta_k)\|g_{k+1}\|^2 + (1-\theta_k)\beta^{DL}{}_k d^T{}_k g_{k+1}$$

$$d^T{}_{k+1} g_{k+1} = \theta_k\left(-\|g_{k+1}\|^2 + \beta^{WYL}{}_k d^T{}_k g_{k+1}\right) + (1-\theta_k)\left(-\|g_{k+1}\|^2 + \beta^{DL}{}_k d^T{}_k g_{k+1}\right)$$

$$d^T{}_{k+1} g_{k+1} = \theta_k d^{WYL}{}_{k+1} g_{k+1} + (1-\theta_k) d^{DL}{}_{k+1} g_{k+1}$$

Firstly, let,$\theta_k = 0$ then $d_{k+1} = d^{DL}{}_{k+1}$

Remember that $d^{DL}{}_{k+1} = -g_{k+1} + \beta^{DL}{}_k d_k$

$$g^T{}_{k+1} d^{DL}{}_{k+1} = -\|g_{k+1}\|^2 + \beta^{DL}{}_k g^T{}_{k+1} d_k$$

$$g^T{}_{k+1} d^{DL}{}_{k+1} = -\|g_{k+1}\|^2 + \left(\frac{(y_k - t s_k)^T g_{k+1}}{d^T{}_k y_k}\right) g^T{}_{k+1} d_k \,,\ t = \frac{2\|y_k\|^2}{s^T{}_k y_k}$$

After substituting the value of t We get:

$$g^T{}_{k+1} d^{DL}{}_{k+1} = -\|g_{k+1}\|^2 + \left(\frac{\left(y_k - \frac{2\|y_k\|^2}{s^T{}_k y_k} s_k\right)^T g_{k+1}}{d^T{}_k y_k}\right) g^T{}_{k+1} d_k$$

$$g^T{}_{k+1} d^{DL}{}_{k+1} = -\|g_{k+1}\|^2 + \left(\frac{g^T{}_{k+1}\left(y_k - \frac{2y^T{}_k y_k}{s^T{}_k y_k} s_k\right)}{d^T{}_k y_k}\right) g^T{}_{k+1} d_k = -\|g_{k+1}\|^2 - \frac{g^T{}_{k+1} y_k}{d^T{}_k y_k} g^T{}_{k+1} d_k$$

$$g^T{}_{k+1} d^{DL}{}_{k+1} = -\|g_{k+1}\|^2 - \|g_{k+1}\|^2 \frac{d^T{}_k y_k}{d^T{}_k y_k}$$

$$g^T{}_{k+1} d^{DL}{}_{k+1} \leq -2\|g_{k+1}\|^2 \leq -C\|g_{k+1}\|^2$$

Now, let $\theta_k = 1$, then $d_{k+1} = d^{WYL}{}_{k+1}$, $d^{WYL}{}_{k+1} = -g_{k+1} + \beta^{WYL}{}_k d_k$

$$g^T{}_{k+1} d^{WYL}{}_{k+1} = -\|g_{k+1}\|^2 + \beta^{WYL}{}_k g^T{}_{k+1} d_k$$

From (12) we get :

$$g^T{}_{k+1} d^{WYL}{}_{k+1} = -\|g_{k+1}\|^2 + \frac{\left(\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} g^T{}_k g_{k-1}\right)}{\|g_{k-1}\|^2} g^T{}_{k+1} d_k$$

The fact that $g^T{}_k g_{k-1} > 0$ means we can get an inequality $0 < \cos\theta_k < 1$ where θ_k is the angle between g_k and g_{k-1}.

Some authors give other discussions and modifications in [3 and 20]. In fact, $g_k - \frac{\|g_k\|}{\|g_{k-1}\|} g_{k-1}$ is not our point at the beginning, our purpose is involving the information of the angle between g_k and g_{k-1} .From this point of view, $\beta^{WYL}{}_k$ has the following form:

$$\beta^{WYL}{}_k = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}(1 - \cos\theta_k) \ \dots\ (17)$$

From (17) We get :

$$g^T{}_{k+1} d^{WYL}{}_{k+1} \leq -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{\|g_k\|^2}(1 - \cos\theta_k)\ \left|g^T{}_{k+1} d_k\right| ,$$

From (4) strong Wolfe-Powell conditions we get :

$$g^T{}_{k+1} d^{WYL}{}_{k+1} \leq -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{\|g_k\|^2}(1 - \cos\theta_k)\left(-\sigma\, g^T{}_k d_k\right)$$

From sufficient descent condition (16):

$$g^T{}_{k+1} d^{WYL}{}_{k+1} \leq -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{\|g_k\|^2}(1 - \cos\theta_k)(\sigma\, h\|g_k\|^2) \,,\ h > 0$$

$$g^T{}_{k+1} d^{WYL}{}_{k+1} \leq -\|g_{k+1}\|^2 + \|g_{k+1}\|^2(1 - \cos\theta_k)(\sigma\, h)$$

$$g^T{}_{k+1} d^{WYL}{}_{k+1} \leq -\left(1 - \sigma\, h(1 - \cos\theta_k)\right)\|g_{k+1}\|^2$$

$$g^T{}_{k+1} d^{WYL}{}_{k+1} \leq -r\|g_{k+1}\|^2 \quad , \quad r = 1 - \sigma\, h(1 - \cos\theta_k)$$

4- Global Convergence

We now show the global convergence result for the new hybrid method. Now we will get to know several hypotheses :

Assumption 1 : A f is function bounded from below on the level set $\Omega=\{f(x) \leq f(x_\circ)\ |x \in R^n\}$, $x_\circ$ is the starting point. In some neighborhood of N of set Ω the objective function is continuously differentiable , An important result for proving the global convergence properties of nonlinear CG algorithms is the famous Zoutendijk condition[7] , that is there exists a constant $A > 0$ such that :

$$\|g(x) - g(y)\| \leq A\|x - y\| , \qquad \forall\, x, y \in N$$

These assumptions imply that there exists a positive constant q such that : $\|g(x)\| \leq q \,, \forall\, x \in N$ ... (18)

Theorem1: Suppose Assumptions 1 hold. Given any iteration of the form (2), where d_k is a descent direction and α_k satisfies (4) and Zoutendijk condition then we have :

$$\sum_{k=0}^{\infty} \frac{\left(g^T{}_k d_k\right)^2}{\|d_k\|^2} < +\infty \,..(19)$$

The proof had been given in [10, 5]. In[8], Gilbert and Nocedal introduced the following important theorem.

Theorem 2: If Assumption 1 holds and $\{x\}$ is generated by the Hybrid CG Algorithm, where d_k is a descent direction, from the strong Wolfe line search. If :

$\sum_{k=0}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} = \infty$ ... (20)

Then $\lim_{k\to\infty} \inf\|g_k\| = 0$ ... (21)

Proof : Suppose that $\|g_k\| \neq 0$ for all k. Then there exists a constant $L > 0$ such that

$\|g_k\| \geq L$ ... (22)

Now we need to simplify my words From (4) and $y_k = (g_{k+1} - g_k)$ we get :

$d^T{}_k\, y_k = d^T{}_k\,(g_{k+1} - g_k) = d^T{}_k\, g_{k+1} - d^T{}_k\, g_k \geq \sigma d^T{}_k\, g_k - d^T{}_k\, g_k \geq c(1-\sigma)\|g_k\|^2$ ... (23)

And from Zoutendijk condition:

$\|y_k\| = \|g_{k+1} - g_k\| \leq A\|x_{k+1} - x_k\| \leq AF$ ... (24)

Where $F = \mathrm{Max}\{\|x-y\|, x, y \in N\}$ is the diameter of the level set N.

From (3) and (12) and (13) we get :

$$|\beta^{FZ}{}_k| \leq |\beta^{DL}{}_k| + |\beta^{WYL}{}_k| \leq \left|\frac{(y_k - tS_k)^T g_{k+1}}{d^T{}_k\, y_k}\right| + \left|\frac{g_k{}^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2}\right|$$

When $t=\frac{2\|y_k\|^2}{s^T{}_k y_k}$ we get :

$$|\beta^{FZ}{}_k| \leq |\beta^{DL}{}_k| + |\beta^{WYL}{}_k| \leq \left|\frac{-g^T{}_{k+1}y_k}{d^T{}_k\, y_k}\right| + \left|\frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|}g^T{}_k\, g_{k-1}}{\|g_{k-1}\|^2}\right|$$

$$|\beta^{FZ}{}_k| \leq \left|\frac{\|g_{k+1}\|\|y_k\|}{c(1-\sigma)\|g_k\|^2}\right| + \left|\frac{\|g_{k+1}\|^2 - \frac{\|g_{k+1}\|}{\|g_k\|}\|g_k\|\|g_{k+1}\|}{\|g_k\|^2}\right|$$

From (18) and (22) and (23) and (24) we get :

$$|\beta^{FZ}{}_k| \leq \left|\frac{qAF}{c(1-\sigma)L^2}\right| + \left|\frac{q^2 - \frac{q}{L}Lq}{L^2}\right|$$

$$|\beta^{FZ}{}_k| \leq \frac{qAF}{c(1-\sigma)L^2} + \frac{q^2 + \frac{q}{L}Lq}{L^2} \leq M \quad ..(25)$$

$\|d_{k+1}\| \leq \|g_{k+1}\| + |\beta^{FZ}{}_k|\|d_k\| \leq \|g_{k+1}\| + M\left\|\frac{S_k}{\alpha_k}\right\|$, $S_k = x_{k+1} - x_k$

$\leq q + M\frac{F}{\gamma} \leq W$, $\gamma > 0$

which implies that :

$\sum_{k\geq 0}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} = \infty$

$$\sum_{k\geq 0}^{\infty} \frac{c^2L^4}{\|d_k\|^2} \leq \sum_{k\geq 0}^{\infty} \frac{\left(\frac{d^T{}_k g_k}{\|g_k\|^2}\right)^2 \|g_k\|^4}{\|d_k\|^2} \leq$$

$$\sum_{k\geq 0}^{\infty} \frac{\frac{(d^T{}_k g_k)^2}{\|g_k\|^4}\|g_k\|^4}{\|d_k\|^2} \leq \sum_{k=0}^{\infty} \frac{(g^T{}_k d_k)^2}{\|d_k\|^2} < +\infty$$

Contradiction we have $\lim_{k\to\infty} \inf\|g_k\| = 0$

5- Numerical Results

In this section, we will discuss the numerical results of the proposed algorithm FZ1 obtained by using the algorithms WYL and DL, as well as the conditional (Wolfe) of a set of test functions in the unconstrained optimization is taken.[19]

To evaluate the performance of this proposed algorithm, (75) test functions were selected, which were included in this study

The functions were chosen for the dimensions, and by comparing the performance of this proposed algorithm with the algorithms Dai–Liao (DL) and Wei–Yao–Liu (WYL) the measure used to stop the repetitions of the algorithms is, the program was written in Fortran language (Based on the program Andrei) (FORTRAN 77) (version 6.6.0), the test functions usually start with the standard starting point and the summary of numerical results is noted in Figures (3.1) and (3.2) And (3.3) and by means of the program (Matlab R2009b), the algorithm evaluation scale was compared based on the method of Dolan and Moore to compare the efficiency of the proposed algorithm with the algorithms of Dai–Liao (DL) and Wei–Yao–Liu (WYL). Defined as a set of test functions and the number of algorithms used. to be $l_{p,s}$ represents the number of times the value of the objective function is found by algorithm s to solve the p problem

$r_{p,s} = \frac{l_{p,s}}{l^*{}_p}$, $l^*{}_p = \min\{l_{p,s} : s \in S\}$

Obviousl $r_{p,s} \geq 1$ for all values p , s If the algorithm fails to solve the problems, then the ratio $r_{p,s}$ Equal to a large number M The efficiency characteristic of the s algorithm is known as a function of the distribution of the compound over the efficiency ratio $r_{p,s}$ $\rho_s(\tau) = \frac{\mathrm{size}\{p\in P: r_{p,s} \leq \tau\}}{n_p}$

Our comparisons includes the following:

1- iter: the number of iteration

2- fg : number of function and gradient evaluations.

3- CPU time

Figs. 1, 2 and 3 shows performance of these methods for solving 70 unconstrained optimization test problems, relative to the iterations (iter), function–gradient evaluations (fg) and CPU time, which are evaluated using the profile of Dolan and More [6]. That is, for each method, we plot the fraction p of problems for which the method is within a factor τ of the best (iter) or (fg) or CPU time.

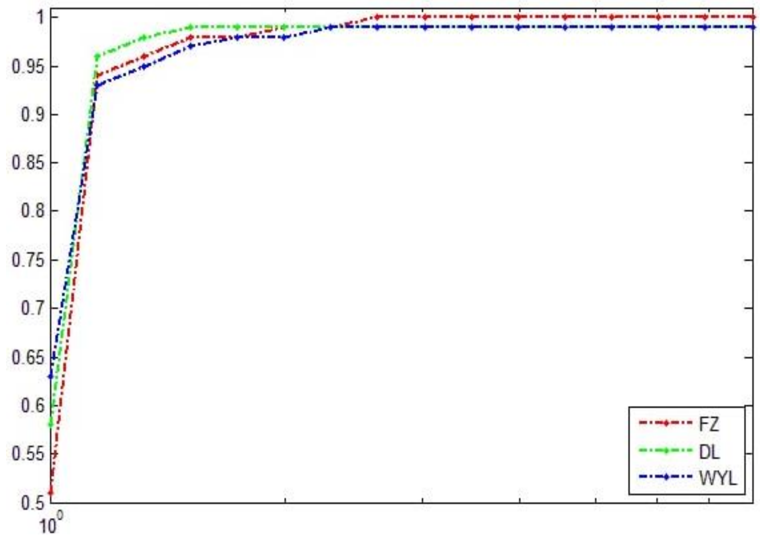

Fig.1: Performance based on iteration

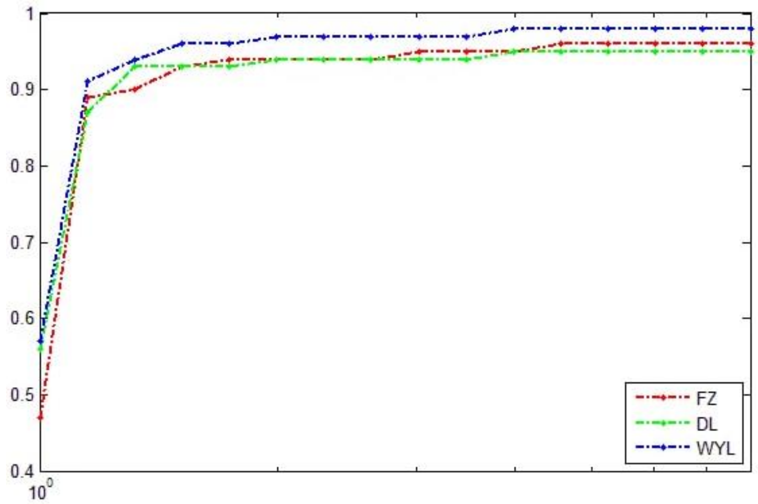

Fig. 2: Comparison based on number of function evaluations for the algorithms DL, WYL and FZ

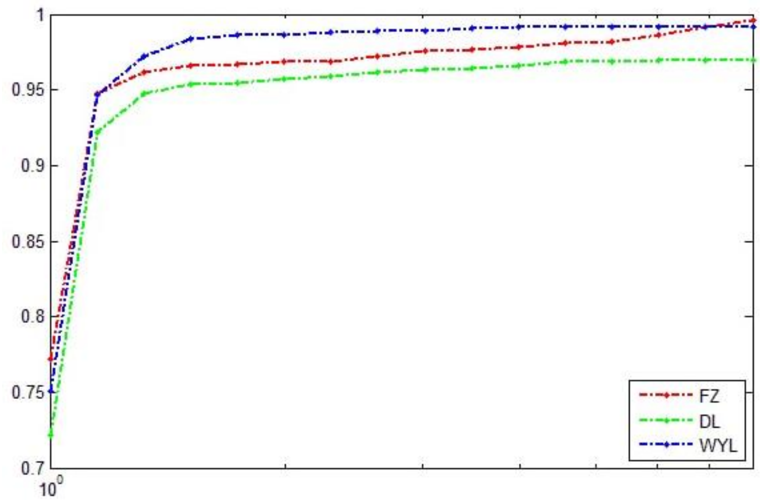

Fig. 3: Performance based on Time

References

[1] B.T. Polyak, The conjugate gradient method in extreme problems, USSR Comp. Math. Math. Phys. 9 (1969) 94–112.
[2] Dai, Y. H., & Liao, L. Z. (2001). New conjugacy conditions and related nonlinear conjugate gradient methods. Applied Mathematics and Optimization, 43(1), 87-101..
[3] Dai, Z., & Wen, F. (2012). Another improved Wei–Yao–Liu nonlinear conjugate gradient method with sufficient descent property. Applied Mathematics and Computation, 218(14), 7421-7430.
[4] Dixon, L., (1975), 'CG-Algorithm Quadratic Termination without Line Search'. Journal of Instituted Mathematics and Application, (15), 9-18.
[5] Delladji, S., Belloufi, M., & Sellami, B. (2021). New hybrid conjugate gradient method as a convex combination of FR and BA methods. Journal of Information and Optimization Sciences, 42(3), 591-602.
[6] Dolan, E. D., & Moré, J. J. (2002). Benchmarking optimization software with performance profiles. Mathematical programming, 91(2), 201-213.
[7] G. Zoutendijk, Nonlinear programming computational methods, in: J. Abadie (Ed.), Integer and Nonlinear Programming, North-Holland, 1970, pp. 37–86
[8] Gilber, J. C., & Nocedal, J. (1992). Global convergence properties of conjugate gradient. SIAM Journal of Optimization, 2, 21-42.
[9] Khalil, A. B. B. O., & HAMEED, N. (2018). New hybrid conjugate gradient method as a convex combination of Liu-Storey and Dixon methods. Journal of Multidisciplinary Modeling and Optimization, 1(2), 91-99.
[10] Lu, J., Yuan, G., & Wang, Z. (2021). A modified Dai–Liao conjugate gradient method for solving unconstrained optimization and image restoration problems. Journal of Applied Mathematics and Computing, 1-23.
[11] M.R. Hestenes, E. Stiefel, Method of conjugate gradient for solving linear equations, J. Res. Nat. Bur. Stand. 49 (1952) 409–436.
[12] R. Fletcher, C. Reeves, Function minimization by conjugate gradients, Compute. J. 7 (1964) 149–154.
[13] R. Fletcher, Practical Method of Optimization, second ed., Unconstrained Optimization, vol. I, Wiley, New York, 1997.
[14] Y. Dai, Y. Yuan, A nonlinear conjugate gradient with a strong global convergence properties, SIAM J. Optim. 10 (2000) 177–182.
[15] Y. Liu, C. Storey, Efficient generalized conjugate gradient algorithms, part 1: theory, J. Optim. Theory Appl. 69 (1992) 129–137.
[16] Yuan, G., Li, T., & Hu, W. (2020). A conjugate gradient algorithm for large-scale nonlinear equations and image restoration problems. Applied Numerical Mathematics, 147, 129-141.
[17] Yuan, G., Meng, Z., & Li, Y. (2016). A modified Hestenes and Stiefel conjugate gradient algorithm for large-scale nonsmooth minimizations and nonlinear equations. Journal of Optimization Theory and Applications, 168(1), 129-152.
[18] Salih, B. M., Abbo, K. K., & Abdullah, Z. M. (2018). Partial Pearson-two (PP2) of quasi newton method for unconstrained optimization. Tikrit Journal of Pure Science, 21(3), 174-179.
[19] Zeyad M Abdullah 2016 New Partial and Memoryless Rank-One and Rank-Two Quasi-Newton Method
[20] Zhang, L. (2009). An improved Wei–Yao–Liu nonlinear conjugate gradient method for optimization computation. Applied Mathematics and computation, 215(6), 2269-2274.

طريقة تهجين جديدة للتدرج المترافق وبصيغة التركيب المحدب لكل من Dia-Laio و Wei–Yao–Liu

زياد محمد عبدالله ، فارس سليم ذياب
قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

في هذا البحث تم تطوير طريقة جديدة من طرائق التدرج المترافق الهجينة . وتعتمد هذه الطريقة في الأساس على تهجين خوارزميات Dia - Laio و Wei–Yao–Liu , وذلك بأستخدام تركيب محدب وشرط الترافق لخط بحث غير مضبوط . الخوارزمية الناتجة تحقق شرط النسب الكافي و ذات تقارب شامل تحت فرضيات معينة .وأشارت النتائج العديدة الى كفاءة هذه الطريقة في حل دوال الاختبار اللاخطية في الامثلية غير المقيدة المعطى .