



## On $\mathfrak{M}$ icro $\alpha$ - generalized closed and $\mathfrak{M}$ icro semi- generalized closed in $\mathfrak{M}$ icro -topological spaces

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### 1. Introduction

One of an essential object in a topological space called closed sets so by using the 'Kuratowski closure axioms' or the axioms of it one can present the topology on sets. N. Levine[1], in (1970), introduce an important definition that provides for, A subset  $\beta$  of  $(X, \mathfrak{F})$  is named generalized closed if  $Cl.(\beta) \subseteq \mathfrak{U}$  whenever  $\beta \subseteq \mathfrak{U}$  for all  $\mathfrak{U}$  is open, P. Bhattacharya and B.K. Lahiri [2], in 1987 present semi-generalized closed sets in topology and in 1995 J. Dontchev [3] study the On generating semi -preopen sets. And it follows in 2002, J.Cao, M. Ganster and I. Reilly [4] described on generalized closed sets. Lellis Thivagar [5] given the concept of  $\mathfrak{N}$ ano-topology in 2013. Also, the authors Bhuvanewari, Ezhilarasi [6] and Thanga Nachiyar [7] introduced on  $\mathfrak{N}$ ano genaeralized semi closed sets and  $\mathfrak{N}$ ano semi -generalized, on  $\mathfrak{N}$ ano A-generalized closed sets and  $\mathfrak{N}$ ano generalized A-closed sets in the spaces of  $\mathfrak{N}$ ano-topology, resp. in 2014. Many authors [8,9] study on  $\mathfrak{N}$ ano Topology and  $\mathfrak{M}$ icro topological spaces. Recently in 2020 the topic of Micro- $\alpha$ -open sets and Micro- $\alpha$ - continuous functions in  $\mathfrak{M}$ icro topological spaces was given by Jasim and Rasheed [10]. This prompts us to continue to develop and

### ABSTRACT

We're going to study a new definitions in this paper that's are  $\mathfrak{M}$ icro  $\alpha$  – generalized closed ,  $\mathfrak{M}$ icro generalized  $\alpha$  – closed,  $\mathfrak{M}$ icro generalized semi – closed and  $\mathfrak{M}$ icro semi – generalized closed. Also, we show the relationships between them in illustration diagram and gives some results and examples.

generalize this topic by presenting some important definitions such as  $\mathfrak{M}$ icro  $\alpha$  – generalized closed ,  $\mathfrak{M}$ icro generalized  $\alpha$  – closed,  $\mathfrak{M}$ icro generalized semi – closed and  $\mathfrak{M}$ icro semi – generalized closed and noting some results on them in a diagram and gives some examples.

### 2. Preliminaries

In principle, we call some important basics and acquaintances.

**Definition2.1[8]:** Let  $X$  a non empty finite set things called the universal and  $\mathfrak{R}$  is the equivalence relation on  $X$  named as the indiscernibility relation elements belonging to the same equivalence class are said to be indiscernible with one another. The pair  $(X, \mathfrak{R})$  is said to be approximation space. Let  $A \subseteq X$ ,

1. The Lower approximation of  $A$  with respect to  $\mathfrak{R}$  the set of all things, which can be for certain classified as  $A$  with respect to  $\mathfrak{R}$  and is defined by  $L_{\mathfrak{R}}(A) = \bigcup_{x \in X} \{\mathfrak{R}(x) : \mathfrak{R}(x) \subseteq A\}$ , Where  $\mathfrak{R}(x)$  denotes the equivalence class determined by  $x$ .

2. The upper approximation of  $A$  with respect to  $\mathfrak{R}$  is these of all things, which can be possibly classified as  $X$  with respect to  $\mathfrak{R}$  and is defined by  $U_{\mathfrak{R}}(A) = \bigcup_{x \in X} \{\mathfrak{R}(x) : \mathfrak{R}(x) \cap A \neq \emptyset\}$ .

3. The boundary region of  $X$  with respect to  $R$  is the set of all things, which can be classified neither as  $A$  nor as not  $A$  with respect to  $\mathfrak{R}$  and is defined by  $B_{\mathfrak{R}}(A) = U_{\mathfrak{R}}(A) - L_{\mathfrak{R}}(A)$ .

**Definition 2.2[8]:** Let  $X$  be the universal,  $\mathfrak{R}$  be an equivalence relation on  $X$  and  $\mathfrak{F}_{\mathfrak{R}}(A) = \{X, \emptyset, L_{\mathfrak{R}}(A), U_{\mathfrak{R}}(A), B_{\mathfrak{R}}(A)\}$  where  $A \subseteq X$ . Then  $\mathfrak{F}_{\mathfrak{R}}(A)$  satisfies The following axioms.

1.  $X$  and  $\emptyset \in \mathfrak{F}_{\mathfrak{R}}(A)$ .
2. The union of the elements of any sub collection of  $\mathfrak{F}_{\mathfrak{R}}(A)$  is in  $\mathfrak{F}_{\mathfrak{R}}(A)$ .
3. The intersection of the elements of any finite sub collection of  $\mathfrak{F}_{\mathfrak{R}}(A)$  is in  $\mathfrak{F}_{\mathfrak{R}}(A)$ .

That is  $T_R(X)$  forms a topology on  $X$  called as Nano topology on  $X$  with respect to  $A$ . We call  $(X, \mathfrak{F}_{\mathfrak{R}}(A))$  as the Nano topological space. The elements of  $\mathfrak{F}_{\mathfrak{R}}(A)$  are called Nano open sets.

**Definition 2.3[9]:** Let  $(X, \mathfrak{F}_{\mathfrak{R}}(A))$  be Nano-topological space then

$\mu_{\mathfrak{R}}(A) = \{N_1 \cup (N_2 \cap \mu) : N_1, N_2 \in \mathfrak{F}_{\mathfrak{R}}(A)\}$  is called 'Micro-topology' of  $\mathfrak{F}_{\mathfrak{R}}(A)$  by  $\mu$  where  $\mu \notin \mathfrak{F}_{\mathfrak{R}}(A)$ .

**Definition 2.4[9]:** A Micro-topology  $\mu_{\mathfrak{R}}(A)$  satisfies the next axioms:

- a)  $X$  and  $\phi \in \mu_{\mathfrak{R}}(A)$ .
- b) The union of the elements of each sub-collection of  $\mu_{\mathfrak{R}}(A)$  is in  $\mu_{\mathfrak{R}}(A)$ .
- c) The intersection of the elements of each finite sub collection of  $\mu_{\mathfrak{R}}(A)$  is in  $\mu_{\mathfrak{R}}(A)$ .

Then  $\mu_{\mathfrak{R}}(A)$  is called Micro topology on  $X$  with respect to  $A$ . The tripartite  $(X, \mathfrak{F}_{\mathfrak{R}}(A), \mu_{\mathfrak{R}}(A))$  is called 'Micro-topological space' and the elements of  $\mu_{\mathfrak{R}}(A)$  are called 'Micro-open' sets and the complement of a 'Micro-open' set is called a 'Micro-closed' set.

**Definition 2.5[1]:** A subset  $\beta$  of  $(X, \mathfrak{F})$  is called generalized closed (shortly g-closed) if  $Cl.(\beta) \subseteq \mathfrak{U}$  and for all  $\beta \subseteq \mathfrak{U}$  for all  $\mathfrak{U}$  is open in  $(X, \mathfrak{F})$ .

**3. Micro  $\alpha$  - GENERALIZED Closed AND Micro semi - GENERALIZED Closed Sets**

Here in this section we will provide definitions and examples and some results obtained

**Definition 3.1[10]:** The Micro- $\alpha$ -closure form a set  $\beta$  of a Micro topological space  $(X, \mathfrak{F}_{\mathfrak{R}}(A), \mu_{\mathfrak{R}}(A))$  is the intersection of all Micro- $\alpha$ -closed sets that contain  $\beta$  and denoted by  $\mu_{\mathfrak{R}}\alpha - Cl.(\beta)$ .

**Definition 3.2:** A subset  $\beta$  of a space  $(X, \mathfrak{F}_{\mathfrak{R}}(A), \mu_{\mathfrak{R}}(A))$  is said to be Micro generalized  $\alpha$ -closed (shortly  $\mu_{\mathfrak{R}}\alpha - closed$ ) if  $\mu_{\mathfrak{R}}\alpha - Cl.(\beta) \subseteq \mathfrak{U}$  and for all  $\beta \subseteq \mathfrak{U}$  for all  $\mathfrak{U}$  is Micro  $\alpha$ -open. The complements of Micro generalized  $\alpha$ -closed is called Micro generalized  $\alpha$ -open.

**Definition 3.3:** A subset  $\beta$  of a space  $(X, \mathfrak{F}_{\mathfrak{R}}(A), \mu_{\mathfrak{R}}(A))$  is said to be Micro  $\alpha$ -generalized closed (shortly  $\mu_{\mathfrak{R}}\alpha g - closed$ ) if  $\mu_{\mathfrak{R}}\alpha - Cl.(\beta) \subseteq \mathfrak{U}$  and for all  $\beta \subseteq \mathfrak{U}$  for all  $\mathfrak{U}$  is Micro open. The complements of Micro  $\alpha$ -generalized closed is called Micro  $\alpha$ -generalized open.

**Example 3.4:** Let's we have the universe set  $X = \{1,2,3,4\}$  with the equivalence relation  $X/\mathfrak{R} =$

$\{\{1\}, \{3\}, \{2,4\}\}$  and  $A = \{2,4\}$  then the Nano-topology is  $\mathfrak{F}_{\mathfrak{R}}(A) = \{\phi, X, \{2,4\}\}$ . Let  $\mu = \{2\}$ , The Micro-topology is  $\mu_{\mathfrak{R}}(A) = \{\phi, X, \{2\}, \{2,4\}\}$ . Sets of closed Micro denoted  $C\mu_{\mathfrak{R}}(A) = \{\phi, X, \{1,3,4\}, \{1,3\}\}$ . We can deduce the collection of all Micro  $\alpha$ -open as:

$$\alpha O(X) = \{\phi, X, \{2\}, \{1,2\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}\}$$

And Micro  $\alpha$ -closed as:

$$\mu_{\mathfrak{R}}\alpha C(X) = \{\phi, X, \{1,3,4\}, \{3,4\}, \{1,4\}, \{1,3\}, \{4\}, \{3\}, \{1\}\}$$

Now, we will take the sets of all power of  $X$  and we check which one of them are  $\mu_{\mathfrak{R}}\alpha$ -g.closed,  $\mu_{\mathfrak{R}}\alpha$ -g.closed and  $\mu_{\mathfrak{R}}\alpha$ -g.closed as in the table:

**Table 1**

P(X)	$\mu_{\mathfrak{R}}\alpha Cl. (A)$	$\mu_{\mathfrak{R}}\alpha Cl. (A)$	$\mu_{\mathfrak{R}}\alpha$ -g.closed	$\mu_{\mathfrak{R}}\alpha$ -g.	$\mu_{\mathfrak{R}}\alpha$ -g.
{1}	{1,3}	{1}	T	T	T
{2}	X	X	F	F	F
{3}	{1,3}	{3}	T	T	T
{4}	{1,3,4}	{4}	F	T	T
{1,2}	X	X	T	T	F
{1,3}	{1,3}	{1,3}	T	T	T
{1,4}	{1,3,4}	{1,4}	T	T	T
{2,3}	X	X	T	T	F
{2,4}	X	X	F	F	F
{3,4}	{1,3,4}	{3,4}	T	T	T
{1,2,3}	X	X	T	T	F
{1,2,4}	X	X	T	T	F
{1,3,4}	{1,3,4}	{1,3,4}	T	T	T
{2,3,4}	X	X	T	T	F
$\phi$	$\phi$	$\phi$	T	T	T
X	X	X	T	T	T

Here we consider three collections:

The collection of

$$\mu_{\mathfrak{R}}\alpha g C(X) = \{\phi, X, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$$

The collection of

$$\mu_{\mathfrak{R}}\alpha g C(X) = \{\phi, X, \{1\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$$

The collection of

$$\mu_{\mathfrak{R}}\alpha g C(X) = \{\phi, X, \{1\}, \{3\}, \{4\}, \{1,3\}, \{1,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}$$

**Result 3.5:** In a space  $(X, \mathfrak{F}_{\mathfrak{R}}(A), \mu_{\mathfrak{R}}(A))$  we have,

- 1) Each  $\mu_{\mathfrak{R}}\alpha$ -g.closed is  $\mu_{\mathfrak{R}}\alpha$ -g.closed.
- 2) Each  $\mu_{\mathfrak{R}}\alpha$ -g.  $\alpha$  closed is  $\mu_{\mathfrak{R}}\alpha$ -g.closed.

**Proof:**

1) Assume that  $\beta \in \mu_{\mathfrak{R}}\alpha g C(X)$  in a space  $(X, \mathfrak{F}_{\mathfrak{R}}(A), \mu_{\mathfrak{R}}(A))$  then  $\mu_{\mathfrak{R}}\alpha Cl.(\beta) \subseteq \mathfrak{U}$  for  $\beta \subseteq \mathfrak{U}$  and for each  $\mathfrak{U}$  is Micro open. Since  $\mu_{\mathfrak{R}}\alpha - Cl.(\beta) \subseteq \mu_{\mathfrak{R}}\alpha Cl.(\beta) \subseteq \mathfrak{U}$  for each  $\mathfrak{U}$  is Micro open, So  $\beta \in \mu_{\mathfrak{R}}\alpha g C(X)$ .

2) Assume that  $\beta \in \mu_{\mathfrak{R}}\alpha g C(X)$  in a space  $(X, \mathfrak{F}_{\mathfrak{R}}(A), \mu_{\mathfrak{R}}(A))$  then  $\mu_{\mathfrak{R}}\alpha - Cl.(\beta) \subseteq \mathfrak{U}$  for  $\beta \subseteq \mathfrak{U}$  for each  $\mathfrak{U}$  is Micro  $\alpha$ -open. Since 'every Micro open is Micro  $\alpha$ -open' then  $\mu_{\mathfrak{R}}\alpha - Cl.(\beta) \subseteq \mathfrak{U}$  for  $\beta \subseteq \mathfrak{U}$  and for each  $\mathfrak{U}$  is Micro open, So  $\beta \in \mu_{\mathfrak{R}}\alpha g C(X)$ .

**Remarks3.6:** From the above result:

- 1) Not all  $\mathbb{Q}.ag.closed$  is  $\mathbb{Q}.g.closed$  like in the previous example the set  $\beta = \{4\} \in \mathbb{Q}agC(X)$  but  $\beta \notin \mathbb{Q}gC(X)$ .
- 2) Also, not all  $\mathbb{Q}.ag.closed$  must be  $\mathbb{Q}.g.\alpha$  closed as in the same example. The set  $N = \{1,2,3\} \in \mathbb{Q}agC(X)$  but not belong to  $\mathbb{Q}g\alpha C(X)$ .
- 3) Both  $\mathbb{Q}gC(X)$  and  $\mathbb{Q}g\alpha C(X)$  are independent as : The set  $\mathcal{H} = \{1,2\} \in \mathbb{Q}gC(X)$  but not in  $\mathbb{Q}g\alpha C(X)$  and  $\beta = \{4\} \in \mathbb{Q}g\alpha C(X)$  but not in  $\mathbb{Q}gC(X)$ .

**Proposition3.7:** If each one of  $A$  and  $B$  are  $\mathbb{Q}.ic.\alpha.g.-closed$  then  $A \cup B$  will be  $\mathbb{Q}.ic.\alpha.g.-closed$ .

**Proof:**

Suppose that  $A$  and  $B$  are  $\mathbb{Q}.ic.\alpha.g.-closed$  then we have  $\mathbb{Q}.ic.\alpha - Cl. (A) \subseteq \mathcal{H}$  for  $A \subseteq \mathcal{H}$  for all  $\mathcal{H}$  is  $\mathbb{Q}.icro \alpha - open$  and  $\mathbb{Q}.ic.\alpha - Cl. (B) \subseteq \mathcal{H}$  and for all  $\mathcal{H} \subseteq \mathcal{U}$  for all  $\mathcal{H}$  is  $\mathbb{Q}.icro open$ . Now, since  $A \subseteq \mathcal{H}$  and  $B \subseteq \mathcal{H}$ , so  $A \cup B \subseteq \mathcal{H}$  s.t.  $\mathcal{H}$  is  $\mathbb{Q}.icro open$ . Then  $\mathbb{Q}.ic.\alpha - Cl. (A \cup B) = \mathbb{Q}.ic.\alpha - Cl. (A) \cup \mathbb{Q}.ic.\alpha - Cl. (B) \subseteq \mathcal{H}$  hence,  $\mathbb{Q}.ic.\alpha - Cl. (A \cup B) \subseteq \mathcal{H}$  whenever  $A \cup B \subseteq \mathcal{H}$  for all  $\mathcal{H}$  is  $\mathbb{Q}.icro open$  implies that  $A \cup B$  be  $\mathbb{Q}.ic.\alpha.g.-closed$ .

**Remark3.8:** The intersection of two  $\mathbb{Q}.ic.\alpha.g.-closed$  in a space  $(X, \mathcal{F}_{\mathcal{R}}(A), \mu_{\mathcal{R}}(A))$  is not  $\mathbb{Q}.ic.\alpha.g.-closed$ , since in example (3.4)  $\{2,3,4\}$ ,  $\{1,2,4\}$  are  $\mathbb{Q}.ic.\alpha.g.-closed$  but  $\{2,4\}$  is not.

**Definition3.9:** The  $\mathbb{Q}.icro-semi-closure$  form a set  $\beta$  of a  $\mathbb{Q}.icro$  topological space  $(X, \mathcal{F}_{\mathcal{R}}(A), \mu_{\mathcal{R}}(A))$  is the intersection of all  $\mathbb{Q}.icro-semi-closed$  sets that contain  $\beta$  and denoted by  $\mathbb{Q}.ic.s - Cl. (\beta)$ .

**Definition3.10:** A subset  $\beta$  in a space  $(X, \mathcal{F}_{\mathcal{R}}(A), \mu_{\mathcal{R}}(A))$  is said to be  $\mathbb{Q}.icro$  generalized semi - closed ( shortly  $\mathbb{Q}.ic.gs - closed$  ) if  $\mathbb{Q}.ic.s - Cl. (\beta) \subseteq \mathcal{H}$  for  $\beta \subseteq \mathcal{H}$  and for all  $\mathcal{H}$  is  $\mathbb{Q}.icro open$ . The complements of  $\mathbb{Q}.icro$  generalized semi - closed is called  $\mathbb{Q}.icro$  generalized semi - open.

**Definition3.11:** A subset  $\beta$  of a space  $(X, \mathcal{F}_{\mathcal{R}}(A), \mu_{\mathcal{R}}(A))$  is said to be  $\mathbb{Q}.icro$  semi - generalized closed ( shortly  $\mathbb{Q}.ic.s.g - closed$  ) if  $\mathbb{Q}.ic.s - Cl. (\beta) \subseteq \mathcal{H}$  for  $\beta \subseteq \mathcal{H}$  for all  $\mathcal{H}$  is  $\mathbb{Q}.icro semi open$ . The complements of  $\mathbb{Q}.icro$  semi - generalized closed is called  $\mathbb{Q}.icro$  semi - generalized open.

**Example3.12:** In example (3.4) let  $\mu = \{3\}$ , The  $\mathbb{Q}.icro - topology$  is  $\mu_{\mathcal{R}}(A) = \{\phi, X, \{3\}, \{2,4\}, \{2,3,4\}\}$ . The  $C\mu_{\mathcal{R}}(A) = \{\phi, X, \{1,2,4\}, \{1,3\}, \{1\}\}$

We can write the collection of all  $\mathbb{Q}.icro$  semi open as:

$$\mathbb{Q}.sO(X) = \{\phi, X, \{3\}, \{1,3\}, \{2,3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}\}$$

And the collection of  $\mathbb{Q}.icro$  semi closed as:  $\mathbb{Q}.sC(X) = \{\phi, X, \{1,2,4\}, \{2,4\}, \{1,3\}, \{3\}, \{1\}\}$

Consider the table:

**Table 2**

P(X)	$\mathbb{Q}.sCl. (A)$	$\mathbb{Q}.g.s$	$\mathbb{Q}.sg.$
{1}	{1}	T	T
{2}	{2,4}	T	T
{3}	{3}	T	T
{4}	{2,4}	T	T
{1,2}	{1,2,4}	T	T
{1,3}	{1,3}	T	T
{1,4}	{1,2,4}	T	T
{2,3}	X	F	F
{2,4}	{2,4}	T	T
{3,4}	X	F	F
{1,2,3}	X	T	T
{1,2,4}	{1,2,4}	T	T
{1,3,4}	X	T	T
{2,3,4}	X	F	F
$\phi$	$\phi$	T	T
X	X	T	T

In this table, the collection of  $\mathbb{Q}.gsC(X)$  and  $\mathbb{Q}.sgC(X)$  are the same.

**Example3.13:** We take the universe set  $X = \{1,2,3,4\}$  with the equivalence relation  $X/\mathcal{R} = \{\{2\}, \{4\}, \{1,3\}\}$  and  $A = \{1,2\}$  then the Nano - topology is  $\mathcal{F}_{\mathcal{R}}(A) = \{\phi, X, \{2\}, \{1,3\}, \{1,2,3\}\}$ . Let  $\mu = \{1\}$ , The  $\mathbb{Q}.icro - topology$  is  $\mu_{\mathcal{R}}(A) = \{\phi, X, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$ . Sets of closed  $\mathbb{Q}.icro$   $C\mu_{\mathcal{R}}(A) = \{\phi, X, \{2,3,4\}, \{1,3,4\}, \{3,4\}, \{2,4\}, \{4\}\}$ . The collection of  $\mathbb{Q}.icro \alpha$ -open is:  $\mathbb{Q}.i\alpha O(X) = \{\phi, X, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}\}$ . And  $\mathbb{Q}.icro \alpha$ -closed as:

$$\mathbb{Q}.i\alpha C(X) = \{\phi, X, \{2,3,4\}, \{1,3,4\}, \{3,4\}, \{2,4\}, \{4\}, \{3\}\}$$

The collection of  $\mathbb{Q}.icro$  semi open is:

$$\mathbb{Q}.sO(X) = \{\phi, X, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,3,4\}, \{1,2,4\}\}$$

And the collection of  $\mathbb{Q}.icro$  semi closed as:

$$\mathbb{Q}.sC(X) = \{\phi, X, \{2,3,4\}, \{1,3,4\}, \{3,4\}, \{2,4\}, \{2,3\}, \{1,3\}, \{4\}, \{2\}, \{3\}\}$$

Now, we will take the sets of all power of  $X$  and we check which one of them are  $\mathbb{Q}.ag.closed$ ,  $\mathbb{Q}.g\alpha.closed$ ,  $\mathbb{Q}.sg.closed$  and  $\mathbb{Q}.g.s$  closed as in the table:

**Table 3**

P(X)	$\mathbb{Q}.sCl. (A)$	$\mathbb{Q}.sg.$	$\mathbb{Q}.g.s$	$\mathbb{Q}.i\alpha Cl. (A)$	$\mathbb{Q}.g\alpha.$	$\mathbb{Q}.ag.$
{1}	{1,3}	F	F	{1,3,4}	F	F
{2}	{2}	T	T	{2,4}	F	F
{3}	{3}	T	T	{3}	T	T
{4}	{4}	T	T	{4}	T	T
{1,2}	X	F	F	X	F	F
{1,3}	{1,3}	T	T	{1,3,4}	F	F
{1,4}	{1,3,4}	F	T	{1,3,4}	F	T
{2,3}	{2,3}	T	T	{2,3,4}	F	F
{2,4}	{2,4}	T	T	{2,4}	T	T
{3,4}	{3,4}	T	T	{3,4}	T	T
{1,2,3}	X	F	F	X	F	F
{1,2,4}	X	F	T	X	F	T
{1,3,4}	{1,3,4}	T	T	{1,3,4}	T	T
{2,3,4}	{2,3,4}	T	T	{2,3,4}	T	T
$\phi$	$\phi$	T	T	$\phi$	T	T
X	X	T	T	X	T	T

Here we consider four collections: The collection of  $\mathbb{M}$ icro semi generalized closed

$$\mathbb{M}sgC(X) = \left\{ \phi, X, \{2\}, \{3\}, \{4\}, \{1,3\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\} \right\}$$

The collection of  $\mathbb{M}$ icro generalized semi closed

$$\mathbb{M}gsC(X) = \left\{ \phi, X, \{2\}, \{3\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\} \right\}$$

The collection of

$$\mathbb{M}g\alpha C(X) = \left\{ \phi, X, \{3\}, \{4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\} \right\}$$

The collection of

$$\mathbb{M}\alpha gC(X) = \left\{ \phi, X, \{3\}, \{4\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\} \right\}$$

**Result3.14:** In a space  $(X, \mathfrak{F}_R(A), \mu_R(A))$  we have,

- 1) Each  $\mathbb{M}$ . g.  $\alpha$  closed is  $\mathbb{M}$ . g. s closed.
- 2) Each  $\mathbb{M}$ .  $\alpha$ g. closed is  $\mathbb{M}$ . g. s closed.
- 3) Each  $\mathbb{M}$ . sg. closed is  $\mathbb{M}$ . g. s closed.

**Proof:**

1) Assume that  $\beta \in \mathbb{M}g\alpha C(X)$  in a space  $(X, \mathfrak{F}_R(A), \mu_R(A))$  then  $\mathbb{M}ic. \alpha Cl. (\beta) \subseteq \mathfrak{U}$  for  $\beta \subseteq \mathfrak{U}$  for each  $\mathfrak{U}$  is  $\mathbb{M}icro \alpha - open$ . Since 'every  $\mathbb{M}icro open$  is  $\mathbb{M}icro \alpha - open$ ' and  $\mathbb{M}ic. s - Cl. (\beta) \subseteq \mathbb{M}ic. \alpha Cl. (\beta) \subseteq \mathfrak{U}$  for each  $\mathfrak{U}$  is  $\mathbb{M}icro open$ , So  $\beta \in \mathbb{M}gsC(X)$ .

2) Assume that  $\beta \in \mathbb{M}\alpha gC(X)$  in a space  $(X, \mathfrak{F}_R(A), \mu_R(A))$  then  $\mathbb{M}ic. \alpha - Cl. (\beta) \subseteq \mathfrak{U}$  for  $\beta \subseteq \mathfrak{U}$  for each  $\mathfrak{U}$  is  $\mathbb{M}icro open$ . Since  $\mathbb{M}ic. s - Cl. (\beta) \subseteq \mathbb{M}ic. \alpha Cl. (\beta) \subseteq \mathfrak{U}$  then  $\mathbb{M}ic. s - Cl. (\beta) \subseteq \mathfrak{U}$  for  $\beta \subseteq \mathfrak{U}$  for each  $\mathfrak{U}$  is  $\mathbb{M}icro open$ , So  $\beta \in \mathbb{M}gsC(X)$ .

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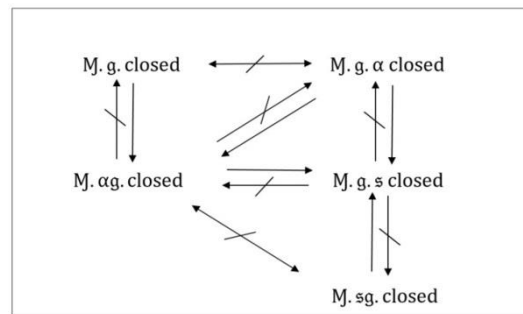
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3) Assume that  $\beta \in \mathbb{M}sgC(X)$  in a space  $(X, \mathfrak{F}_R(A), \mu_R(A))$  then  $\mathbb{M}ic. sCl. (\beta) \subseteq \mathfrak{U}$  for  $\beta \subseteq \mathfrak{U}$  for each  $\mathfrak{U}$  is  $\mathbb{M}icro s - open$ . Since 'every  $\mathbb{M}icro open$  is  $\mathbb{M}icro s - open$ ' and  $\mathbb{M}ic. s - Cl. (\beta) \subseteq \mathfrak{U}$  for  $\beta \subseteq \mathfrak{U}$  for each  $\mathfrak{U}$  is  $\mathbb{M}icro open$ , So  $\beta \in \mathbb{M}gsC(X)$ .

**Remarks3.15:** From the above result:

- 1) Not all  $\mathbb{M}$ . gs. closed be  $\mathbb{M}$ . g.  $\alpha$  closed like in the previous example the set  $\beta = \{2\} \in \mathbb{M}gsC(X)$  but  $\beta \notin \mathbb{M}g\alpha C(X)$ .
  - 2) Also, not all  $\mathbb{M}$ . gs. closed must be  $\mathbb{M}$ .  $\alpha$ g. closed as in the same example. The set  $\beta = \{2,3\} \in \mathbb{M}gsC(X)$  but not belong to  $\mathbb{M}\alpha gC(X)$ .
  - 3) Also, not all  $\mathbb{M}$ . gs. closed must be  $\mathbb{M}$ . sg. closed as in the same example. The set  $\beta = \{1,2,4\} \in \mathbb{M}gsC(X)$  but not belong to  $\mathbb{M}sgC(X)$ .
  - 4) Both  $\mathbb{M}sgC(X)$  and  $\mathbb{M}\alpha gC(X)$  are independent as : The set  $\mathcal{H} = \{2\} \in \mathbb{M}sgC(X)$  but not in  $\mathbb{M}\alpha gC(X)$  and  $\beta = \{1,4\} \in \mathbb{M}\alpha gC(X)$  but not in  $\mathbb{M}sgC(X)$ .
- Finally, we obtain the following diagram from the previous results



**Diagram 1**

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## حول مايكرو ألفا المعممة المغلقة وشبه المعممة المغلقة في الفضاءات التوبولوجية الدقيقة

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### الملخص

سنقوم بعمل تعريفات جديدة في هذه الورقة وهي مايكرو ألفا معممة مغلقة ، مايكرو معممة ألفا مغلقة ، مايكرو معممة شبه مغلقة و مايكرو شبه معممة مغلقة. كما نعرض العلاقات بينهما في مخطط توضيحي ونعطي بعض النتائج والأمثلة.