Tikrit Journal of Pure Science Vol. 28 (3) 2023

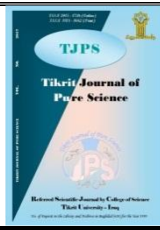
TJPS

https://doi.org10.25130//tjps.v28i3.1431

Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j



ζ-Open Sets and ζ-Continuity

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ARTICLE INFO.

Article history:

-Received: 4 / 9 / 2022

-Received in revised form: 18 / 9 / 2022

-Accepted: 23 / 10 / 2022

-Final Proofreading: 7 / 5 / 2023

-Available online: 25 / 6 / 2023

Keywords: ζ-Open Set, ζ-continuous map, ζ-Topological Space, ζ-Separation axioms

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ABSTRACT

In this work, the concept of ζ-Open Set was introduced as a generalization of open set. Where it is shown that (X, τ_ζ) is a Topological Space.

Also, we study ζ -Interior, ζ -Closure, Continuity Function. At last basics Properties of them are given.

المجموعة المفتوحة من النوع ζ و الاستمرارية عليها

صباح عبد الله تمر ، طه حميد جاسم

كلية علوم الحاسوب و الرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

في هذا البحث، تم تقديم مفهوم المجموعة المفتوحة من النوع ζ كتعميم للمجموعة المفتوحة. حيث تبين أن (X, τ_ζ) هي فضاء طوبولوجي.

كما ندرس وظيفة الاستمرارية من النوع ζ – مجموعة النقاط الداخلية من النوع ζ – الانغلاق من النوع ζ. و أخيرا سوف نناقش بديهيات الفصل لهذا النوع من المجموعات المفتوحة

1. Introduction and Basic concepts

In this work, we present some basic concepts in the topological space, such as the interior which the scientist studied S.-M. Jung and D. Nam year 2019 [1] and Closure which the scientist studied by N. Levine at year 1961 [2], as well as the continuous functions that the scientist studied by Cenap Ozel at 2021[3], Then the Separation axioms T_0, T_1, T_2 was studied by M. Bharti at 2019 [4]. Finally, some of the theorems and properties that we will generalize open set to the ζ -Open set type.

The definition of topological space is given in [5] as If X ≠ ∅, set, τ is a family of subsets $\tau \subseteq P(X)$ is named to be **topology** on X if the following holds:

1- $X, \emptyset \in \tau$.

2- If $S, U \in \tau \Rightarrow S \cap U \in \tau$

3- If $S_i \in \tau \Rightarrow \cup_i S_i \in \tau$

We called a pair (X,τ) **Topological space**.

In [2] the interior set definition is given as: the topological space (X, τ), if S ⊆ X. The interior of S denoted as union of every open set whose contained

https://doi.org10.25130//tjps.v28i3.1431

in S symbolized by Int (S) or S° i.e., the largest open set that's contained in S.

Also, the closure set was given in **[**3] as: Assume (X, τ) be a topological space, $S \subseteq X$. The **closure** of S is defined as the smallest closed set whose contained in S denoted by $Cl\,(S)$ *or* $\bar{S}$.

i.e., the smallest closed set that contained.

Definition 1.1[6]

Let $f:(X,\tau) \to (Y,\ddot{\tau})$ be a function where $(X,\tau), (Y,\ddot{\tau})$ are topological spaces. The function f is names continuous function if $f^{-1}(S) \in X$ for all open set in Y.

A function $f:(X,\tau) \to (Y,\ddot{\tau})$ is named an open if $f(S)$ is an open set in Y, whenever $S \in \tau$.

A function $f:(X,\tau) \to (Y,\ddot{\tau})$ is named closed if $f(S)$ is a closed set in Y, whenever S is a closed set in X.

Definition 1.2 [4], [7],[8]

A topological space (W, τ) is called a **T_0-space** if for any pair of different points of W, there is at least one open set which includes one of them but not the other.

A topological space (W, τ) is named a **T_1-space** if for any pair of different points ẋ, ȳ in W, there is two open sets $S, T \in \tau$,s.t. ẋ $\in S$, ȳ $\notin S$ and ẋ $\notin T$, ȳ $\in T$.

Let (W, τ) a topological space is named **T_2-space** or Hausdorff space if for each pair of different points can be separated by disjoint open set.

2. Main results

In this section, some new class of sets called Zeta-Open sets was introduced (briefly ζ –open) with some properties.

At last the concept of continuity with some characterization were introduced too.

Definition 2.1

Let (X, τ) be topological space and $S \subseteq X$. Then is S is ζ **-open set**

If $S \cap \bar{U} \neq \emptyset\ \forall z \in S\ \exists$ open set U content z s.t $\emptyset \neq U \neq X$ when $\emptyset \neq S \neq X$

And $S \cap \bar{U} = \begin{cases} \emptyset & if \quad S = \emptyset \\ X & if \quad S = X = U \end{cases}$

The set of all ζ –open is denoted by is ζ –O(X).

Example 2.2:

Let X={ ả, ᶀ,ç} ,τ={ ∅,{ ả },{ ả, ᶀ },X} and τ^c={X,{ ᶀ, ç },{ ç }, ∅} then is ζ –O(X).

Let S=X, ∅ is ζ -Open set {from def.}.

Let S={ả} is ζ -Open set

Let S={ ᶀ } is ζ -Open set

Let S={ ç } is not ζ -Open set

Let S={ ả , ᶀ } is ζ -Open set

Let S={ ả , ç } is not ζ -Open set

Let S={ ᶀ , ç } is not ζ -Open set

∴ τ_ζ= {X, ∅,{ ả },{ ᶀ },{ ả , ᶀ }}

Theorem 2.3:

Every open set is ζ-open set and the opposite isn't true.

Proof: Let S be a open set if $S \neq \emptyset$ or X Then is proof done .

Assume $\emptyset \neq S \neq$ X and z ∈ S

Put G=S , $\bar{S} = \bar{G} \Rightarrow S \cap \bar{G} = S \neq \emptyset$.

Example 2.4:

Let X={ ả, ᶀ,ç} ,τ={ ∅,{ ả },{ ả, ᶀ },X} and τ^c={X,{ ᶀ, ç },{ ç }, ∅} then is ζ –O(X)

∴ ζ **-O(X)={X, ∅,{ ả },{ ᶀ },{ ả, ᶀ }}, { ᶀ } is ζ-open set but not open set**

Theorem 2.5:

(X, ζ -O(X)) is topological space.

1- $\emptyset$, X $\in \zeta - O(X)$ (by def. $\zeta - O(X)$)

2- Let $S_\alpha \in \zeta - O(X)$ and z ∈∪ $S_\alpha \Rightarrow \alpha^* \in \Lambda$ s.t z∈ $S_{\alpha^*}\ \exists\ G_{\alpha^*}, z \in G_{\alpha^*}$

$S_{\alpha^*} \cap \bar{G}_{\alpha^*} \neq \emptyset \Rightarrow \bigcup_{\alpha\in\Lambda} (S_\alpha \cap \bar{G}_{\alpha^*}) \neq \emptyset$

∴ $\bigcup_{\alpha\in\Lambda} S_\alpha \cap \bigcup_{\alpha\in\Lambda} \bar{G}_{\alpha^*} \neq \emptyset$

3- Let $S_1, S_2 \in$ ζ-O(X), *To prove* $S_1 \cap S_2 \in \zeta -$ O(X) $s.t\ z \in S_1 \cap S_2$

4- $\Rightarrow z \in S_1, z \in S_2 \exists\ open\ G_1, G_2\ contain\ z$ s.t $S_i \cap \bar{G}_i \neq \emptyset\ \forall\ i = 1,2$

∴ $S_1 \cap S_2 \cap \bar{G}_1 \cap \bar{G}_2 \neq \emptyset$ (since z ∈ $S_i \cap \bar{G}_i$)

Corollary 2.6 :

(X, τ_ζ) finer then (X, τ) .

Proof : By**(Theorem 3.3)** Every open set is ζ-open set.

Definition 2.7:

Let (X, τ_ζ) be a topological space and $S \subseteq X$. A point $z \in S$ is called an ζ **- interior point** of S and is denoted by S^o_z or Int$_\zeta$ (S). i.e.,

$Int_\zeta\,(S) = \{z \in S : \exists\, U \in \tau_\zeta;\ z \in U \subseteq S\}$

$z \in Int_\zeta(S) \Leftrightarrow \exists\, U \in \tau_\zeta;\ z \in U \subseteq S$

Example 2.8:

Let X={ ả,ᶀ,ç} ,τ={X,{ ả },{ ả, ç }, ∅} and τ^c={∅,{ ᶀ,c},{ ᶀ },X} then is

τ_ζ ={X,{ ả },{ ç },{ ả, ç }, ∅}. then is Int$_\zeta$(S) . Let S=X

Let S=X,∅ ⟹ Int$_\zeta$ (S) = X,∅.

Let S={ ả } ⟹ Int$_\zeta$ (S) = {ả}.

Let S={ ᶀ } ⟹ Int$_\zeta$ (S) = ∅.

Let S={ ç } ⟹ Int$_\zeta$ (S) = { ç }.

Let S={ ả, ᶀ }⟹ Int$_\zeta$ (S) = { ả }.

Let S= {ả, ç } ⟹ Int$_\zeta$ (S) = { ả, ç }.

Let S= {ᶀ, ç} ⟹ Int$_\zeta$ (S) = {ç}.

Definition 2.9:

Let (X,τ_ζ) be topological space and $S_\zeta \subseteq$ X. The **ζ-closure** of sets S_ζ is $S_\zeta \cup S_\zeta^{`}$ and denoted by $\bar{S_\zeta}$ or $cl_\zeta$(S) .i.e. $\bar{S_\zeta} = S_\zeta \cup S_\zeta^{`}$

Example 2.10:

Let X={ ả,ᶀ,ç} ,τ={X,{ ᶀ },{ ᶀ, ç }, ∅} and τ^c={∅,{ ᶀ, ç },{ ç },X} then is

τ_ζ={X,{ ᶀ },{ ç },{ ᶀ, ç }, ∅} and ζ -c(x) ={∅ , { ả, ç },{ ả, ᶀ },{ ả },X} then is b_ζ(S). Let S={X}

Let S=X,∅ ⟹ cl_ζ(S)= X,∅

Let S={ ả } ⟹ cl_ζ(S) = { ả, ç }.

Let S={ ᶀ } ⟹ cl_ζ(S)={ ả, ᶀ }.

Let S={ ç } ⟹ cl_ζ(S)={ ả, ç }.

Let S={ ả, ᶀ }⟹ cl_ζ(S)={a, ᶀ }.

https://doi.org10.25130//tjps.v28i3.1431

Let S={ å, ç } $\Longrightarrow cl_{\zeta}$(S)={ å, ç }.

Let S={ ḅ, ç } $\Longrightarrow cl_{\zeta}$(S)={X}.

Definition 2.11:

Let (X,τ_{ζ}) and (Y,$\ddot{\tau_{\zeta}}$) be topological space and f: (X,τ_{ζ}) →(Y,$\ddot{\tau_{\zeta}}$)

The function f is called **ζ-continuous** if the inverts image for any ζ-open set Y is an ζ-open set in X .i.e.

f: (X,τ_{ζ})→ (Y,$\ddot{\tau_{\zeta}}$)is ζ-continuous $\Leftrightarrow f^{-1}(U) \in \tau_{\zeta}$ $\forall\, U \in \ddot{\tau_{\zeta}}$

and the function f is called ζ-dis-continuous if there exist an ζ-open set in Y. but inverts image is not ζ-open set in X.i.e.

f is ζ-dis-continuous $\Leftrightarrow U \in \ddot{\tau_{\zeta}} \wedge f^{-1}(U) \notin \tau_{\zeta}$

Example 2.12 :

Let X={1,2,3,4} ,τ_x={X, ∅,{1,2}} τ_x^c={X, ∅,{3,4} and Y={ å,ḅ,ç} ,τ_y={Y,∅,{ ḅ, ç }} , τ_y^c={∅,Y,{ å }} then is

τ_{ζ}={X, ∅,{1,2},{1},{2}} $\ddot{\tau_{\zeta}}$={X, ∅,{ å },{ ḅ },{ å, ḅ }}

Define h: (X,τ_{ζ})→(Y,$\ddot{\tau_{\zeta}}$) ;$h(1)$ = å , $f(2)$= ḅ ,$f(3)$ = $f(4)$= ç

Thus h ζ- continuous

Theorem 2.13 :

if f : (X,τ_{ζ}) →(Y,$\ddot{\tau_{\zeta}}$) and g : (Y,$\ddot{\tau_{\zeta}}$)→(M,$\breve{\tau_{\zeta}}$) are both ζ -continuous function , then the composition gof : (X,τ_{ζ}) →(M,$\breve{\tau_{\zeta}}$) is ζ-continuous

proof : Let V∈ $\breve{\tau_{\zeta}} \Longrightarrow g^{-1}(V) \in \ddot{\tau_{\zeta}}$ (since g is ζ-continuous)

notes that $g^{-1}(V) \subseteq$ Y

$\Longrightarrow f^{-1}(g^{-1}(V)) \in \tau_{\zeta}$ (since is f is ζ-continuous)

$\Longrightarrow (f^{-1}og^{-1})(V) \in \tau_{\zeta}$ (by composition is f is ζ-continuous)

$\Longrightarrow (fog)^{-1}(V) \in \tau_{\zeta}$ (since $(gof)^{-1}$=$f^{-1}og^{-1}$)

∴ gof is ζ-continuous. the figure below clear this theorem .

3- ζ-Seperation axioms

Some new class of separation axioms were introduced in this section with their relations.

Definition 3.1 :

A topological space(X,τ_{ζ}) is ζ-**T0 – space**, if for each pair of distinct points x̧, ȳ ∈ X, there is either an ζ -open set containing x̧ but not ȳ or an ζ -open set containing ȳ but not x̧.

Example 3.2 :

Let X={ å,ḅ, ç } ,τ_x={X, ∅,{ å },{ å, ḅ }} , τ_x^c={∅,X,{ ḅ, ç },{ ç }}

then is τ_{ζ}={X, ∅,{ ḅ },{ ç },{ ḅ, ç }} . is (X, τ_{ζ}) is ζ-T_0 – space.

Example 3.3 :

Let (X, τ_x) be topological space

X={a,ḅ,ç} ,τ_x={X, ∅,{ å, ḅ }} , τ_x^c={X,∅,{ ç }}

then is τ_{ζ} ={X, ∅,{ å },{ ḅ },{ å, ḅ }} and let ζ-τ_x ={X, ∅,{ å, ḅ }} then is ζ- **T_0 – space**

∴ **(X , ζ-τ_x)** is not ζ- **T_0 – space**

Because å, ḅ ∈ X and ∄ ζ -open set containing å but not ḅ.

Theorem 3.4 :

The property of being a ζ-T₀ – space is a hereditary property.

Proof :

Let (X, τ_{ζ}) ζ-T0 – space and (W, $\ddot{\tau_{\zeta}}$) subspace of X , to prove (W, $\ddot{\tau_{\zeta}}$) is ζ-T0 – space

Let x̧, ȳ ∈ W ; x̧ ≠ ȳ ⟹ x̧, ȳ ∈ X (since W ⊆X)

∵X is ζ-T0 – space $\Longrightarrow \exists\, U \in \tau_{\zeta}$; (x̧ ∈ U ∧ ȳ ∉ U) ∨ (x̧ ∉ U ∧ ȳ ∈ U)

⟹ U ∉W∈(W, $\ddot{\tau_{\zeta}}$)

(by def. of $\ddot{\tau_{\zeta}}$)

; (x̧ ∈ U∩W ∧ ȳ ∉ U∩W) ∨ (x̧ ∉ U∩W ∧ ȳ ∈ U∩W)

∴ (W, $\ddot{\tau_{\zeta}}$) is ζ-T0 – space.

Definition 3.5 :

Let (X, τ_{ζ}) be a topological space. Then is the space (X, τ_{ζ}) is called

ζ-T_1 – Space if for each pair of distinct points x̧, ȳ ∈ X, there exists an ζ- open set in X containing x̧ but not ȳ, and an ζ -open set in X containing ȳ but not x̧.

Example 3.6:

Let X={å,ḅ,ç} ,τ_x={X, ∅,{å},{å,ḅ},{å, ç }} , τ_x^c={X,∅,{ ḅ, ç },{å, ç },{ ḅ }} then is τ_{ζ} ={X, ∅,{ å },{ ḅ },{ ç },{ å, ḅ },{ å, ç }}is ζ- **T_1 – space**.

Example 3.7:

Let X={ å, ḅ, ç} ,τ_x={X, ∅,{ å, ḅ }} , τ_x^c={X,∅,{ ç }}

then τ_{ζ} ={X, ∅,{å},{ ḅ },{å, ḅ }}

∴ (X, τ_{ζ}) is not ζ- T_1 – space

Because $a, c \in X$ and ∄ two ζ − open sets U, P s. t.

a ∈ U , a ∉ P and c ∉ U , c ∈ P

Remark 3.8 :

every ζ- T_1 – space is ζ-T_0-spase (i.e. ζ- T_1 ⟹ ζ-T_0). But the reverse implications doesn't hold (i.e. ζ- T_0 ⇏ ζ-T_1). As in the example below :

Example 3.9 :

Let X={å,ḅ, ç } ,τ_x={X, ∅,{ ḅ,ç},{ ç }} , τ_x^c={X,∅,{ å },{ å, ḅ }} then, τ_{ζ}={X, ∅,{ ḅ },{ ç },{ ḅ, ç }}, then

(X, τ_{ζ}) is ζ- T_0 – space. But doesn't ζ- T_1 – space.

Definition 3.10 : Let (X, τ_{ζ}) be a topological space, $z \in X$ and $S \subseteq X$. S is said to be ζ − neighborhood for a point z if there exist a ζ − open set U contains z which is contain in S denoted by $nbhd_{\zeta}$

Theorem 3.11 :

(X, τ_{ζ}) is ζ-T_1 – Space if { x̧ } is ζ- closed ∀ x̧ ∈ X.

i.e., (X, τ_{ζ}) is ζ-T_1 – Space if every singleton set in X is ζ- closed.

Proof : (⟹) Suppose that X is ζ-T_1 – Space, to prove {x̧ } ζ- closed ∀ x̧ ∈X

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i.e., X – { ᶍ } ζ- open set, we must prove X–{ ᶍ } contains a $nbhd_ζ$ ∀ ȳ ∈X–{ ᶍ }

Let y ∈ X – { ᶍ } ⇒ ᶍ ≠ ȳ

∴ X is ζ-T_1 – Space ⇒ ∃ U, V_y ∈ ζ-$\tau_ζ$; (ᶍ ∈ U ∧ ȳ ∉ U) ∧ (ᶍ ∉ $V_{ȳ}$ ∧ ȳ ∈ $V_{ȳ}$)

⇒ ȳ ∈ $V_{ȳ}$ ∧ ᶍ ∉ $V_{ȳ}$

⇒ { ᶍ }∩ $V_{ȳ}$ = ϕ

⇒ $V_{ȳ}$ ⊆ X – { ᶍ } ∧ ȳ ∈ $V_{ȳ}$

⇒ $V_{ȳ}$ ⊆ X – { ᶍ } ∀ ȳ ∈ X – { ᶍ }

∴ X – { ᶍ } contains a $nbhd_ζ$ ∀ ȳ ∈ X – { ᶍ }.

∴ X – { ᶍ } ζ- open set ⇒ { ᶍ } ζ- closed ∀ ᶍ ∈ X.

(⇐) suppose that { ᶍ } ζ- closed ∀ ᶍ ∈ X, to prove X is ζ-T_1 – Space

Let ᶍ, ȳ ∈ X ; ᶍ ≠ ȳ ⇒ { ᶍ }, { ȳ } are ζ- closed sets

⇒ X – { ᶍ }, X – { ȳ } are ζ- open sets

Say U = X – { ȳ }, V = X – { ᶍ } ⇒ (ᶍ ∈ U ∧ ȳ ∉ U) ∧ (ᶍ ∉ V ∧ ȳ ∈ V)

∴ (X, $\tau_ζ$) is ζ-T_1 – Space.

Definition 3.12 :

Let (X, $\tau_ζ$) be a topological space. Then is the space (X, $\tau_ζ$) is called a **ζ-T_2 – space or Hausdorff space.**let ᶍ, ȳ ∈ X, for each pair of distinct points, there exist ζ -open sets U and V such that ᶍ ∈ U, ȳ ∈ V, and $U ∩ V = \emptyset$ i.e., X is ζ-T2 – space ⇔ ∀ ᶍ, ȳ ∈ X ; ᶍ ≠ ȳ∃ $U, V \in \tau_ζ$; (ᶍ ∈ U ∧ ȳ ∈ V), $U ∩ V = \emptyset$

Example 3.13 :

Let X={ å, ḅ, ç } ,τ_x={X, ∅,{ å,ḅ },{ ḅ, ç },{ ḅ }} and τ_x^c={X,∅,{ ç },{ å },{ å, ḅ}}

so $\tau_ζ$ ={X, ∅,{ å },{ ḅ },{ ç },{ å, ḅ },{ ḅ, ç }} . Then (X,$\tau_ζ$) is **ζ- T_2 – space**

Example 3.14 :

Let X={å,ḅ, ç } ,τ_x={X, ∅,{ å, ḅ }} and τ_x^c={X,∅,{ç}} So, $\tau_ζ$ ={X, ∅,{å},{ ḅ },{å, ḅ }} is **ζ- T_2 – space**. But (X, $\tau_ζ$) is not **ζ- T_2 – space**

Remark 3.15 :

every ζ- T_2 – space is ζ-T_1-spese (i.e. ζ- T_2 ⟹ ζ-T_1). But the reverse implications don't hold (i.e. ζ- T_1 ⇏ ζ-T_2). As in the example below

Example 3.16 :

Is (Ɲ, $\tau_{ζ(cof)}$) ζ-T1 – space.

Let ᶍ, ȳ ∈ Ɲ ; ᶍ≠ȳ, ∃ U = Ɲ − {ᶍ} ∈ $\tau_{ζ(cof)}$

V = Ɲ − {ȳ} ∈ $\tau_{ζ(cof)}$ (since U^c = (Ɲ − {ᶍ})^c={ᶍ}

V^c = (Ɲ − {ȳ})^c={ȳ} finite set by define of $\tau_{ζ(cof)}$)

⇒(ȳ ∈ U= Ɲ − {ᶍ} ∧ ᶍ ∉ U) ∧(ȳ ∉ V= Ɲ − {ᶍ} ∧ ᶍ ∈ V)

But (Ɲ, $\tau_{ζ(cof)}$) is not ζ-T_2 – space if ᶍ≠ȳ, ∃ U = Ɲ − {ᶍ} ∈ $\tau_{ζ(cof)}$

V = Ɲ − {ȳ} ∈ $\tau_{ζ(cof)}$, but $U ∩ V \neq \emptyset$

Theorem 3.17:

Let (X, $\tau_ζ$) be a ζ-T_2 – space if the diagonal Δ = {(ᶍ, ᶍ) ∈ X × X ; ᶍ ∈ X} is a ζ-closed subset of the product X × X.

Proof : (⟹) Suppose that X is ζ-T_2 – space, to prove Δ ζ-closed in X × X

i.e., X × X – Δ ζ-open set, we must prove X × X– Δ contains a $nbhd_ζ$ ∀ (ᶍ, ȳ) ∈ X × X – Δ Let (ᶍ, ȳ) ∈ X × X – Δ⟹ (ᶍ, ȳ) ∉ Δ (def. of deference)

⟹ ᶍ ≠ ȳ (since Δ has equal coordinate)

∵ X is a ζ-T_2 – space ⟹ ∃ $U, V \in \tau_ζ$, ∩$V = \emptyset$, (ᶍ ∈ U ∧ ȳ ∈ V)

⟹ $U × V \in \beta_{X×X} \subseteq \tau_{ζ(X×X)}$ (by def. product space)

⟹ $U × V$ ζ-open set in X × X and

$U × V$ ⊆ X × X – Δ ∧ (ᶍ, ȳ) ∈ $U × V$ (since ∩$V = \emptyset$)

Since, if $U × V$ ⊈ X × X – Δ ⟹ ∃ (ᶍ, ᶍ) ∈ Δ ⟹ ᶍ∈ U ∧ x ∈ V C!!

∴ X × X – Δ contains a $nbhd_ζ$ ∀ ᶍ ∈ X × X – Δ

⟹ X × X – Δ ∈ $\beta_{x×x}$

⟹ Δ ζ-closed in X×X

(⟸) Suppose that Δ ζ-closed in X × X, to prove X is ζ-T2 – space

Let ᶍ, ȳ ∈ X ; ᶍ ≠ ȳ ⟹ (ᶍ, ȳ) ∉ Δ (by def. of Δ)

⟹ (ᶍ, ȳ) ∈ $Δ^c$= X × X – Δ

∴ Δ ζ -closed set ⟹ X × X –Δ ζ -open set

⟹ ∃$U × V$; $U, V \in \tau_ζ$ ∧ (ᶍ, ȳ) ∈ $U × V$, $U × V$ ⊆ X × X – Δ , ᶍ ∈ U, ȳ ∈ V

⟹ $U × V ∩ Δ = \emptyset$ (i.e., ∄ element in $U × V$ has equal coordinate)

⟹ $U × V = \emptyset$

⟹ (X, $\tau_ζ$) is ζ-T_2 – space.

https://doi.org10.25130//tjps.v28i3.1431

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