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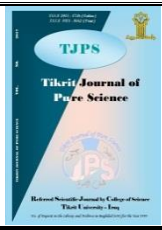

A study of the fixed and variable capacitances numerical stability in the diffusion and load equation

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ABSTRACT

In this paper A study of the fixed and variable capacitances numerical stability in the diffusion and load equation. in two cases , the first when the amplitude is constant the solution is stable , and the second is variable the solution in this case is stable conditionally.

1- Introduction

That any system if it exist in a situation , it is stable if external in fluences do not affect its condition , for example the solar system depends on time and that the planets revolve around the sun in harmony when a celestial body enters the solar system , the system is not affected because the original stable is not affected by influences or disturbances , that the system is stable The study of numerical stability in this type explains many properties of stability in physics , chemistry ,biology and fluid mechanics , it shows that any disturbance or small confusion in the physical system leads to far reaching results. The kind of stability helps to know the future scale of the physical system, it also shows the effect of in stability in the system[1].

In (1986) scientist test Hickernell, Yortsos, linear stability of miscible displacement processesin a transcendental region in the absence of scattering and diffusion [2].

In (1986) a test (Howes)the stability of the constant solutions to the equations of heat change and diffusion special terms have been used on the occurrence of interaction and diffusion , to prove the stability of the time lager[3].

In (1998) suggest the worlds fortunate and kurizhi, way it work and impressively steady point stability in the traditional moton of the sensitive system [7].

In (1978) study the world maginu, the stability of the necessary solution of this type by using liapunov method[5] .

In (1980) study the worlds conley and smoller , how to use anumber of topological concepts to get information minute about for timely solutions to equation pregnancy and spreed [8].

2- Mathematical model

One of the diffusion equations is the interaction equation of load and diffusion: [4].

$$\frac{\partial u\,[x,t]}{\partial t} = D\,\frac{\partial^2 u[x,t]}{\partial x^2} - V\,\frac{\partial u\,[x,t]}{\partial x} \quad \dots (1)$$

And if that

$X \in (0\,,1)\ ,\ u(x\,,0\,) = \phi(x)\ \ ,u\,(1,t) = u\,(0,t) = 0\ ,t > 0\,,D\,,V\ > 0$

D: represents the diffusion coefficient and V: represents the reaction coefficient[5] .

3- Dimensional transformations of the diffusion and convection equation

The equation for load and diffusion can be converted into an equation (1) to the non – dimensional state in$(x\,,t)$, we will need transfers:

$$X=\frac{x}{L}\quad,\quad T=\frac{t}{L^2}$$

Using this transformation, we can get:

$$\frac{\partial u\,[x\,,t]}{\partial x}=\frac{\partial u\,[x\,,t]}{\partial x}\frac{\partial X}{\partial x}=\frac{1}{L}\frac{\partial u\,[x\,,t]}{\partial X}$$

$$\frac{\partial^2 u[x\,,t]}{\partial x^2}=\frac{1}{L^2}\frac{\partial^2 u\,[x\,,t]}{\partial x^2}$$

$$\frac{\partial u\,[x\,,t]}{\partial t}=\frac{\partial\, u[x\,,t]}{\partial T}\frac{\partial T}{\partial t}=\frac{1}{L^2}\frac{\partial u\,[x\,,t]}{\partial T}$$

When substituting the above quantities in an equation (1), we get

$$\frac{1}{L^2}\frac{\partial u\,[x\,,t]}{\partial T}=\frac{D}{L^2}\frac{\partial^2 u\,[x\,,t]}{\partial X^2}-\frac{V}{L}\frac{\partial u[x\,,t]}{\partial T}$$

$$\Rightarrow \frac{\partial\, u\,[x\,,t]}{\partial T}=D\,\frac{\partial^2 u[x\,,t]}{\partial X^2}-VL\,\frac{\partial u\,[x\,,t]}{\partial X}\ ....(2)$$

$0<X<1\quad,\ D\,,V>0\quad,\ u\,[0\,,t]=u\,[1\,,\ t]=0$

And that equation (2) represented the general non-dimensional form of the equation for load and diffusion

4- Stability analysis when the amplitude is constant

The basic idea of this method it is assuming the solution is as follows: [1] $u[X,T]=u_1\,[X]+u_2[X,T]\ .....(3)$

And that $u_1[X]$ represents the necessary solution to the equation (2) while $u_2[X,T]$ represents the confusion to which the system is exposed

$$u_2\,[x\,,t]=A\,e^{ik\,[x-cT]}\ ....(4)$$

$i=\sqrt{-1}\ ,\ C=C_1+i\,C_2\ ,\quad K>0\ ,\ A>0$

A: represent the amplitude of the wave, K: represent the wave number, c: wave speed

When replacing the equation (3)in equation (2) , we get

$$\frac{\partial u_1[x\,,t]}{\partial T}=D\left[\frac{\partial^2 u_1(x)}{\partial X^2}+\frac{\partial^2 u_2(x\,,t)}{\partial X^2}\right]-VL\left[\frac{\partial u_1(x)}{\partial x}+\frac{\partial u_2(x\,,t)}{\partial x}\right]$$

When we separate the temporal and temporal cases, we can get:

$$\frac{\partial u_2[x\,,t]}{\partial T}=D\,\frac{\partial^2 u_2[x\,,t]}{\partial X^2}-VL\,\frac{\partial u_2[x\,,t]}{\partial x}\ ..(5)$$

$$D\,\frac{\partial^2 u_1(x)}{\partial X^2}-VL\,\frac{\partial u_1(x)}{\partial x}=0\ \ldots(6)$$

And that the equation (5) represent the time state, and that the equation (6) represent temporal state , and the solution is as follows : [6]

$$u_1(x)=\frac{A\sqrt{B}}{\sqrt{B+4D}}\,e^{-\left[\frac{[x-v]^2}{B+4D}\right]}\ \ldots\ (7)$$

And that A,B information constant and optional

When replacing the equation (4) in equation (5) we can get:

$$-ikAe^{ik[x-cT]}=-D\,k^2\,A\,e^{ik\,[x-cT]}-VL\,Aik\,e^{2ik\,[x-cT]}$$

$$\Rightarrow\ -ik=-Dk-iVL$$

$$\Rightarrow\ -ic_1+c_2=-Dk-iVL$$

$$c_1=VL\quad,\ c_2=-Dk$$

Since the values D,K always be positive , so $c_2<0$ thus , the system is stable under any circumstances >

5- Stability analysis when the capacitance is variable

Numerical stability will be studied in this section analytically when the capacitance is variable, and the equation (4) turns into:

$$u_2\,(X\,,T)=A\,(X)\,e^{ik\,[x-cT]}\ \ldots\ \ldots(8)$$

When replacing the equation (8) in equation (5) produce:

$$ickA(X)e^{ik[x-cT]}=DA^{``}(X)\,e^{ik\,[x-cT]}+2iDk\,A^{`}(X)e^{ik[x-cT]}-Dk^2A(X)\,e^{ik[x-cT]}+VLikA\,(X)\ e^{ik\,[x-cT]}+V\,L\,A^{`}\ (X)\,e^{ik[x-cT]}$$

$$\Rightarrow\ c_2\,KA(X)=D\,A^{``}(X)-Dk^2\,A(X)+VLA^{`}(X)$$

$$\Rightarrow\ A^{``}(X)+\frac{VL}{D}A^{`}(X)-\left[\frac{C_2k}{D}-k\right]A(X)=0\ \ldots(9)$$

And the general solution to the equation (9) and with boundary condition (2) is : [1]

$$A\,(X)=e^{-\frac{x}{2}}\,[\cos\beta\,x+B\,\sin\beta\,x]\ldots\ (10)$$

And that

$$\beta=\frac{VL}{2D}\sqrt{1-4\,c_2k}$$

And $A\,,B$ optional constant, when using boundary conditions we can get: $A\,(X)=0\ \Rightarrow\ A=0$, $A(1)=0\quad\Rightarrow\ B=0$

$AX=0$

This is an un finished solution

And to determine the value (B), we can change $B\neq 0$ if:

$B\,\sin\beta=0\quad\Rightarrow\ \beta=n\,\pi\quad,\quad n=1\,,2\,,3\,,\ldots\ldots$

But

$$\beta=\frac{V}{2D}\sqrt{1-4c_2k}=n\,\pi$$

If

$$\frac{VL}{2D}\sqrt{1-4c_2k}=n\,\pi$$

$$\sqrt{1-4c_2k}=\frac{2Dn\pi}{VL}\qquad\text{(by squaring both sides)}$$

$$1-4c_2k=\left[\frac{2D\,n\,\pi}{VL}\right]^2$$

$$C_2=\frac{1}{4K}\left(1-\left[\frac{2D\,n\,\pi}{VL}\right]^2\right)\ ....(11)$$

And the equation (11) there are three cases including:

1- The solution is not stable if $\left[\frac{2D\,n\,\pi}{VL}\right]^2<1$ so $c_2>0$

2- The solution is stable if $\left[\frac{2D\,n\,\pi}{VL}\right]^2>1$ so $c_2<0$

3- If it was$\left[\frac{2D\,n\,\pi}{VL}\right]^2=1$ so c = 0 , in order to obtain the stability curve , we can take the lowest subjective value and this is at n=1 , L= 0,1,2,…,10

$$L=\frac{2D\pi}{V}\ldots\ (12)$$

It is an equation (12) can we get:

$$L\,V=2D\pi\ \Rightarrow\ D=\frac{L\,V}{2\pi}\ \ldots.\ (13)$$

And when applying an equation (13), L = 0,1,2,3,…,10 we get results below in the following table :

Table (1-1) show the stability of the diffusion

L	V=0.2 D=?	V= 0.4 D=?	V= 0.6 D=?
0	0.0	0.0	0.0
1	0.0318	0.0637	0.0955
2	0.0637	0.1274	0.1911
3	0.0955	0.1911	0.2866
4	0.1274	0.2548	0.3822
5	0.1592	0.3185	0.4777
6	0.1911	0.3822	0.5732
7	0.2229	0.4459	0.6688
8	0.2548	0.5096	0.7643
9	0.2866	0.5732	0.8599
10	0.3185	0.6369	0.9554

From the above table we note , region the stability increase when the diffusion coefficient is large , that is the stability is clear from the diffusion equation ,and this mean that the diffusion equation is stable (absolutely stable)

It is an equation (5-2-14-1) , we can get the equation

$$V = \frac{2D\pi}{L} \quad \text{..... (14)}$$

And when applying an equation (14) , L = 0,1,2,3,…,10 we get results below in the following table :

Table (1-2): show the stability of the load factor

L	(D=0.2) V=?	(D= 0.4) V=?	D =(0.6) V=?
1	1.256	2.512	3.768
2	0.628	1.256	1.884
3	0.4187	0. 8373	1.256
4	0. 314	0.628	0. 942
5	0. 2512	0. 5024	0. 7536
6	0. 2093	0. 4187	0. 628
7	0. 1794	0. 3589	0. 5383
8	0. 157	0. 314	0. 471
9	0. 1396	0. 2791	0. 4187
10	0. 1256	0. 2512	0. 3768

From the above table, we can see region the stability decrease, when load factor values are small, that is the stability of the load factor is clear

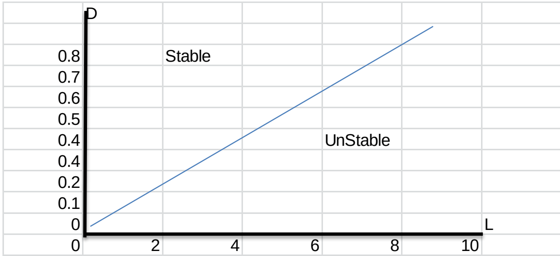

Fig. (1-3): show the stability areas the load equation (12) when V= 0.2 , 0.4 , 0.6

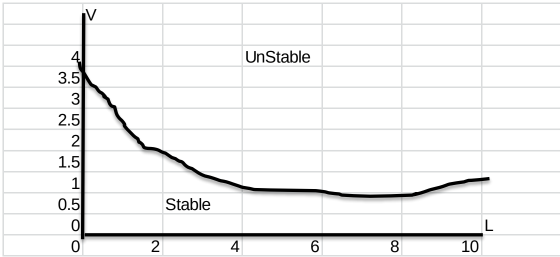

Fig. (1-4): show the stability areas the diffusion equation (14) when D= 0.2 , 0.4 , 0.6

Conclusions

We can conclude from the above figures and tables that :

1- The solution is not stable if $\left[\frac{2D\,n\,\pi}{VL}\right]^2 < 1$ so $c_2 > 0$

2- The solution is stable if $\left[\frac{2D\,n\,\pi}{VL}\right]^2 > 1$ so $c_2 < 0$

3- The equilibrium stability curve is $L = \frac{2D\pi}{V}$

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دراسة الاستقرارية العددية للسعة ثابتة والمتغيرة في معادلة الانتشار والحمل

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الملخص

في هذا البحث قمنا بدراسة الاستقرارية العددية في معادلة الانتشار والحمل في حالتين الاولى عندما تكون السعة ثابتة , والثانية عندما تكون السعة متغيرة , وتبين عند دراسة الاستقرارية العددية للسعة الثابتة انها تكون مستقرة دائما , بينما السعة المتغيرة تكون ذات استقرارية مشروطة .