IASJ
المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science
*ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)*

Journal Homepage: http://tjps.tu.edu.iq/index.php/j

# On Some Types of Matrices for Fan Plane Graph and Their Dual

**Haneen Mohammed Adil, Israa Munir Tawfik**
*Department of Mathematics, College of Education for Women, Tikrit University, Tikrit , Iraq*
https://doi.org/10.25130/tjps.v28i2.1341

**ARTICLE INFO.**

**Article history:**
**-Received:** 4 / 10 / 2022
**-Accepted:** 19 / 10 / 2022
**-Available online:** 26 / 4 / 2023

**Keywords:** Adjacency matrix, Incidence matrix, Degree matrix, Plane graph and Dual graph.

**Corresponding Author:**
**Name:** Haneen Mohammed Adil
**E-mail:** Haneen.adill@st.tu.edu.iq
Israa.tawfik@tu.edu.iq
**Tel:**



## ABSTRACT

This work aims to discuss the adjacency matrices, Incidence matrix and Degree matrix of some types plane graphs we usually used them, as complete graphs, cycle graph,…,ect. To find the dual of graph and transformation of the graph and their dual for some theorems to prove general cases.

## 1. Introduction

A graph $G=(V,E)$ where $V$ is a finite set of vertices denoted by $V(G)$ and $E$ is a finite set of edges denoted by $E(G)$ When we say $V(G)$ we mean the set of vertices $v_1,\dots,v_n$ and $E(G)$ we mean the set of edges of $G$ $e_1,\dots,e_n$. This paper works with the relationship between graph theory, liner algebra and topological surfers, by using matrices and sphere. We are working here on the plane graph, which is a graph that can be drawn in the plane where there are no intersections between its edges.

Presented interesting Arabic book on graph theory, in which define the adjacency matrix and incidence matrix[1]. Studied the graph and its basic concepts[2]. Presented graph theory, basis concepts and some matrices[3].

Euler's discovered the fundamental theorem in graph theory, also studied [4] the graph with its matrices, it also presented known graph and referred to the incidence and adjacency matrix. the basic concepts of the graph and presented some types of graph[5].

The closest study to our paper [6] it worked with plane graphs and their dual for some graphs such as the cobweb graph and the $P_{r,s}$ graph. And [7] also referred to the matrices related to the theory of the graph and the various relationships that presented some of the known graph and the adjacency and incidence matrix. This research consists of three items basic concepts, definitions and theorems.

## 2. Basic concepts:

In this item of paper, we will present the main definitions and proofs for this topic of our paper.

**Defined2-1:** Let $G=(V,E)$ be a graph and $M(G)=[m_{ij}]$ symmetric matrix of the order $n*n$, then it is said that $M(G)$ is adjacency matrix if $m_{ij}$ is the number of edges connecting the vertex $v_i$ to the vertex $v_j$ [7].

**Defined2-2**: Let $G=(V,E)$ be a graph and $I(G)=[i_{ij}]$ order of matrix $n*m$ where $n$ the number of the vertices and $m$ the number of the edges then said to be $I(G)$ Incidence matrix if $i_{ij}=1$ when the vertex $v_i$ is one end of the edge $e_i$ and $i_{ij}=0$ otherwise [7].

**Defined2-3**: if $G=(V,E)$ is a graph, where $G$ that does not contain any intersections between its edges or it can be avoided it said to be plane graph otherwise be non-plane graph [8].

**Theorem2-1:**
Let $G = (V, E)$ be a plane graph has $n$ vertices, $m$ edges and $f$ faces where
$n - m + f = 2$ **[9]**.
**Theorem2-2:**
Let $G = (V, E)$ be a plane graph only $K_5$, $K_{3,3}$ is non plane graph **[9]**.
Now we have to define the dual graph of a graph $G$ ,denoted by $G^*$.The vertices in $G^*$ are denoted by $f$ where we choose a point inside each face to represent us a vertex, the number of edges in $G^*$ is the same number in $G$. Where the $n^*$ in $G^*$ is the number of vertices, $e$ in $G$ $e^*$ in $G^*$ is the number of edges but regarding $v$ in $G$ corresponds to $f$ in $G^*$[9].
**Defines:**
**1**-Let $G = (V, E)$ be a star graph where it has two vertices of one degree connected to an edge forming the shortest path so that each vertex is one-degree has a degree of 2, where there $v_n$ is the cut-vertex and center vertex of the graph is said to be **Fan graph** and denoted by $\boldsymbol{F_n}$ [5] as in the figure (1):

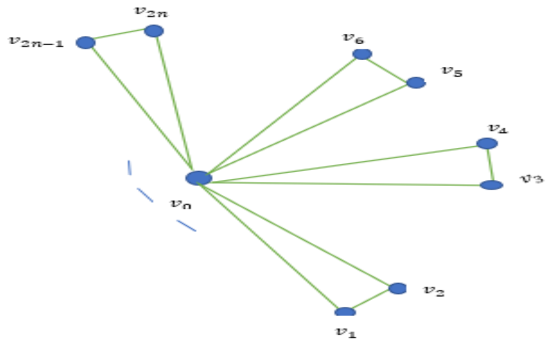

**Fig. 1: Fan graph**

2- Let $G = (V, E)$ be a fan graph $n \geq 3$ set of vertices $v_1, \dots, v_n$ added a new set of vertices $u_1, \dots, u_n$ where between each pair of vertices of the fan$v_i, v_{i+1}$ there is an adjacent $u_i$, $2n$ of the edges where $v_n$ is cut-vertex , said to be **Cog-Fan Graph** denoted by $\boldsymbol{F_n^c}$ **[5]** as in the figure(2):

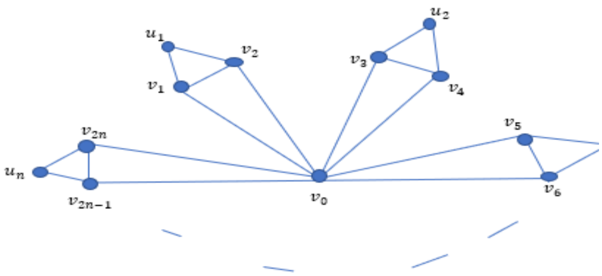

**Fig. 2: Cog-Fan Graph**

3- Let $G = (V, E)$ be a half-wheel graph deleted an edge $W_n - \{e\}$ where $v_n$ is center vertex is said to be **Hand Fan Graph** and denoted by $\boldsymbol{HF_n}$ [9] as in the figure(3):

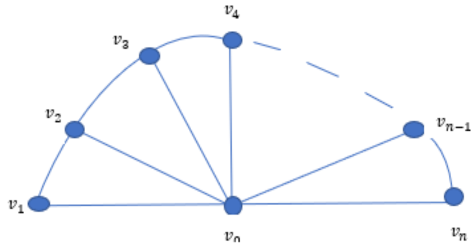

**Fig. 3: Hand Fan Graph**

4-Let $G = (V, E)$ be a hand fan graph $n \geq 3$ set of vertices $v_1, \dots, v_n$ added a new set of vertices $u_1, \dots, u_n$ where between each pair of vertices of the hand fan$v_i, v_{i+1}$ there is an adjacent $u_i$, $2n$ of the edges where $v_n$ is center vertex , said to be **Cog-Hand Fan Graph** denoted by $\boldsymbol{HF_{n+1}}$ **[5]** as in the figure(4):

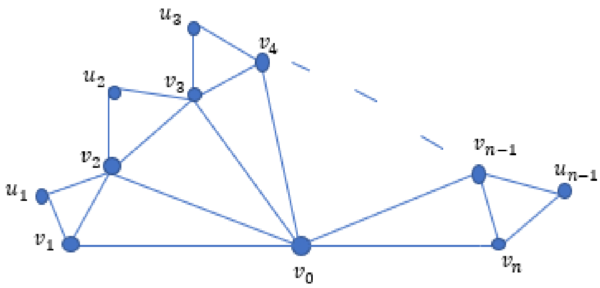

**Fig. 4: Cog-Hand Fan Graph**

After knowing the previous graphs, we can say show by wheel graph of the order exact $n$ after adding a new set of vertices to it as show in the definition below, we have the fan graph all of its regions (three and four edges) and from here said to be cycle fan graph.
5- Let $G = (V, E)$ be a wheel graph the number of vertices $v_1, \dots, v_n$ is added to it a new set of vertices $n - 1$ ,where between each $v_i, v_{i+1}$ there is $u_i$ adjacent to them and the number of added edges $2n - 2$, $v_n$ center vertex , said to be **Cycle fan graph** and denoted by $\boldsymbol{CF_n}$ as in the figure(5):

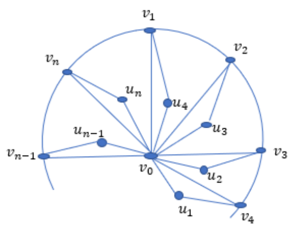

**Fig. 5: Cycle fan graph**

## 3. The matrices of some plan graphs and their dual:

**Theorem3.1:** Let $= F_n$ , $n \geq 3$ then the following statements are true:
1. $M(G) = n * n$ square binary matrix of zero diameter.
2. $I(G) = n * m$ binary matrix, where $m$ is the number of edges.

3. $D(G) = n * n$ diagonal square matrix, where $\deg(v_0) = n - 1$, $\deg(v_i) = 2$ when $i = 1, \ldots, n$.

Proof:

1. $G = F_n$ , where $G$ is a simple graph and the matrix of the simple graph is a zero-diameter binary matrix.
2. It is obvious.
3. $G$ is a simple graph, $\exists v_0 \in G$ where $v_0$ is cut-vertex adjacent all vertices we get $\deg(v_0) = n - 1$, $v_i \in G$ when $i = 1, \ldots, n$ the vertices $v_{i+n}, v_n$ adjacent with $v_0$ we get $\deg(v_i) = 2$.

**Theorem3.2:** Let $G^* = F_n^*$ dual of $F_n$, then the following statements are true:

1. $M(G^*) = n^* * n^*$ a square matrix of zero diameter, all elements are zeros except for the last row and column $i_{nn}, j_{nn} = 3$.
2. $I(G^*) = n^* * m$ binary matrix where $n^*$ number vertices of $F_n^*$.
3. $D(G^*) = n^* * n^*$ diagonal square matrix, where $\deg(f_n) = 3(n - 1)$ , $\deg(f_i) = 3$ when $i = 1, \ldots, n - 1$.

Proof:

1. $G^* = F_n^*$ is a non-simple graph that does not contain a loop we get $Diag(M) = 0$, Each face in $G$ is surrounded by a $3e$ where in $G^*$ is parallel edge and another edge, where $\forall f_i \in G$ when $i = 1, \ldots, n - 1$ adjacent with only $f_n$ we get $i_{nn}, j_{nn}$ in $M(G^*)$ equal 3.
2. It is obvious.
3. It is obvious.

**Theorem3.3:** Let $= F_n^c$ , $n \geq 3$ then the following statements are true:

1. $M(G) = n * n$ square binary matrix of zero diameter.
2. $I(G) = n * m$ binary matrix, where $m$ is the number of edges.
3. $D(G) = n * n$ diagonal square matrix, where $\deg(v_i) = 3$ , $\deg(v_0) = \mathrm{n} - 1$ when $i = 1, \ldots, n$.

Proof:

1. It is obvious.
2. It is obvious.
3. $D(G)$ a diagonal matrix by definition. Since $G = F_n^c$ a simple graph, $\exists v_0 \in V(G)$ where $v_0$ is a cut vertex $\deg(v_0) = n - 1$,and $v_i, v_{i+1} \in V(G)$ vertices adjacent to each other and with a new set of vertices added $u_i$ so $\deg(v_i) = 3$ when $i = 1, \ldots, n$ . As for the added set of vertices $\deg(u_i) = 2$ when $i = 1, \ldots, n$.

**Theorem3.4:** Let $G^* = F_n^{c*}$ dual of $F_n^c$, then the following statements are true:

1. $M(G^*) = n^* * n^*$ square binary matrix of zero diameter, where $M(G^*) = 0,1$ except for the last row and column $i_{nn}, j_{nn} = 2$.
2. $I(G^*) = n^* * m$ binary matrix where $n^*$ number vertices of $F_n^{c*}$.
3. $D(G^*) = n^* * n^*$ diagonal square matrix, where $\deg(f_i) = 3$ , $\deg(f_n) = 2\,[\deg(v_n)]$ when $i = 1, \ldots, n - 1$.

Proof:

1. It is obvious.
2. It is obvious.
3. $D(G^*)$ a diagonal matrix by definition. From the definition of $G^*$ where the outer face of $G$is $f_n$, the vertex of $G$ is twice as The degree of the cut vertex (central) $v_0$ in $G$. As for the interior faces of $G$ since we have parallel edges $\exists e \in E(G)$ and $\forall f \in F(G)$ has 3e So in $G^*$ it becomes $\deg(f_i) = 3$ when $i = 1, \ldots, n - 1$

**Theorem3.5:** Let $= HF_n$, $n \geq 3$ then the following statements are true:

1. $M(G) = n * n$ square binary matrix of zero diameter.
2. $I(G) = n * m$ binary matrix, where $m$ is the number of edges.
3. $D(G) = n * n$ diagonal square matrix, where $\deg(v_0) = n - 1$ , $\deg(v_i) = 3$ , $\deg(v_n) = \deg(v_1) = 2$ when $i = 2, \ldots, n - 1$.

**Proof:**

**1.** It is obvious.
**2.** It is obvious.
**3.** $D(G^*)$ a diagonal matrix by definition. $G = HF_n$, $v_0$ be a center vertex where $\deg(v_0) = n - 1$ and $\deg(v_i) = 3$ when $i = 2, \ldots, n - 1$. Where $v_i$ adjacent to each other and with the center vertex $v_0$ be her degree 3 as for $v_i, v_{n-1}$ adjacent to with two vertex ,where $\deg(v_n) = \deg(v_1) = 2$ , since the $HF_n$ a wheel from which the edge that connects the primary vertex to the final vertex has been removed, then these two vertices has a degree of 2.

**Theorem3.6:** Let $G^* = HF_n^*$ dual of $HF_n$, then the following statements are true:

1. $M(G^*) = n^* * n^*$ square binary matrix of zero diameter, where $M(G^*) = 0,1,2$
2. $I(G^*) = n^* * m$ binary matrix where $n^*$ number vertices of dual.
3. $D(G^*) = n^* * n^*$ diagonal square matrix, $D(G^*)$ elements are odd number, where $\deg(f_i) = 3, \deg(f_n) = n$, when $i = 1, \ldots, n - 1$.

Proof:

**1.** It is obvious.
**2.** It is obvious.
3. $D(G^*)$ a diagonal matrix by definition. From definition $G^*$ , each $G^*$ has one outer face, where $f_n$ is the outer face of G whose degree in $G^*$ is $\deg(f_n) = n$ and also $G^*$ has n inner face where $\deg(f_i) = 3$ when $i = 1, \ldots, n - 1$.

**Theorem3.7:** Let $= HF_{n+1}$, $n \geq 3$ then the following statements are true:

1. $M(G) = n * n$ square binary matrix of zero diameter.
2. $I(G) = n * m$ binary matrix, where $m$ is the number of edges.
3. $D(G) = n * n$ diagonal square matrix, where $\deg(v_0) = n - 1$ , $\deg(v_1) = \deg(v_n) = 3, \deg(v_i) = 5$ when $i = 2, \ldots, n - 1$ and $\deg(v_i) = 2, \deg(u_i) = 2$ when $i = 1, \ldots, n$.

Proof:

**1.** It is obvious.
**2.** It is obvious.

3. $D(G^*)$ a diagonal matrix by definition. Since $G = HF_{n+1}$ has a center vertex $v_0$ where $\deg(v_0) = n-1$ as for the non-adjacent vertices with $v_0$ which $u_i$ represents its degree 2 where when $i = 1, \dots, n$.
Also, the degree of the vertices of the graph that adjacent to the previous set and center vertex $\deg(v_1) = \deg(v_n) = 3$ when $i = 1, n$ and $\deg(v_i) = 2, \deg(u_i) = 2$ when $i = 1, \dots, n$.

**Theorem3.8:** Let $G^* = HF^*_{n+1}$ dual of $HF_{n+1}$, then the following statements are true:
1. $M(G^*) = n^* * n^*$ square binary matrix of zero diameter, where $M(G^*) = 0,1,2$
2. $I(G^*) = n^* * m$ binary matrix where $n^*$ number vertices of $HF^*_{n+1}$.
3. $D(G^*) = n^* * n^*$ diagonal square matrix, $\deg(f_n) = 2n$, $\deg(f_i) = 3$ when $i = 1, \dots, n-1$.

Proof:
**1.** It is obvious.
**2.** It is obvious.
3. $D(G^*)$ a diagonal matrix by definition. From definition $G^*$, each $G^*$ has one outer face $f_n$ where $\deg(f_n) = 2n$, which adjacent all the inner faces of the graph represented by $f_i$ when $i = 1, \dots, n-1$. Whose degree 3 because all inner faces are triangles.

**Theorem3.9:** Let $= CF_n$, $n \geq 3$ then the following statements are true:
1. $M(G) = n * n$ square binary matrix of zero diameter.
2. $I(G) = n * m$ binary matrix, where $m$ is the number of edges.
3. $D(G) = n * n$ diagonal square matrix, where elements are odd number.

Proof:
1. It is obvious.
2. It is obvious.
3. $D(G)$ a diagonal matrix by definition. Since $= CF_n$, the sub graph for $G$is $S_n$ , $F_n$ .where $v_0$ central vertex wheredeg$(v_0) = n-1$ . $v_i \in$ G always where $\deg(v_i) = 4$ when $i = 1, \dots, n$ when $v_i$ is a vertex in the wheel and when adding $u_{i-1}$ becomes $\deg(v_i) = 4$ and for the added vertices of $G$ it is $\deg(u_i) =$ 2when $i = 1, \dots, n$.

**Theorem3.10:** Let $G^* = CF^*_n$ dual of $CF_n$, then the following statement are true:
1. $M(G^*) = n^* * n^*$ square binary matrix of zero diameter, where $M(G^*) = 0,1,2$ .
2. $I(G^*) = n^* * m$ binary matrix where $n^*$ number vertices of dual.
3. $D(G^*) = n^* * n^*$ diagonal square matrix, $D(G^*) = 3,4$.

Proof:
**1.** It is obvious.
**2.** It is obvious.
3. $D(G^*)$ a diagonal matrix by definition From. we notice that there are parallel edges in $G$ where $\exists f_i \in F(G^*)$, is one of the two ends of $e$ where $\deg(f_i) = 4$ for some $i < n$ and the rest of the faces $\deg(f_i) = 3$ for some $i < n$.

## References


[1] Aziz, A. A.(1983).Introduction to data theory, Republic of Iraq, Ministry of Higher Education and Scientific Research.

[2] Tawfik, I.M.(2013).Links and graphs, Ph.D. thesis, Plymouth University.

[3] Fasfous, M.W.(2017).A Study of Graph Theory with Matrix, Representation, Science, Mathematics, M.Sc., thesis, Paestine University.

[4] Jakhlab, S. A.(2017). d-distance coloring for some graphs, Faculty of Sciences, Mathematics. Tishreen University.

[5] Mohammed, H. N.(2018).On Schultz and Modified Schultz polynomials of some Cog-special graphs and two Gutman`s operations, M.Sc.thesis, Mosul University, College of Computer Sciences, Mosul,Iraq.

[6] Zaben, R. A. and Tawfik, I. M. (2021).The Matrices of Some of the Plane Graphs and Their Dual. *Journal of Physics: Conference Series*, **1897** (**012077**):1-8. IOP Publishing.

[7] Islam, M.et al.(2022).Matrix Representation of Graph Theory with Different Operations. *Jornal of Mathematics*,**18(1)**: 08-27.

[8] Trudeau, R. J.(2013). Introduction to graph theory, Courier Corporation.

[9] Harary, F.(1971).Graph Theory, Addison-weley.


# بعض أنواع المصفوفات لبيانات المروحة المستوية وثنائيها


حنين محمد عادل ، اسراء منير توفيق

*قسم الرياضيات ، كلية التربية للبنات ، جامعة تكريت ، تكريت ، العراق*



**الملخص**

يهدف هذا العمل الى مناقشة المصفوفات التجاور والوقوع والدرجة لبعض أنواع من البيانات المستوية المستخدمة بالعادة مثل البيان الكامل والدائرة ... الخ. وكذلك إيجاد ثنائي البيان وتحويل البيان وثنائيه الى بعض المبرهنات مع ثنائيها.