On strongly faintly M-θ-i-continuous functions in Bi-Supra Topological Space

Taha H. Jasim[1] , Reem O. Rasheed[2]
[1] Department of Mathematics, college of computer science and Mathematics, University of Tikrit , Tikrit , Iraq
[2] Department of Mathematics , college of Education for pure sciences , University of Tikrit , Tikrit , Iraq

Abstract.
In this paper we introduces a new definition , called i- open and via this definition we introduce class of topological concepts(μ-θ-i-open set, μ-θ-i-closed, strong faintly μ-θ-continuity, strong μ-θ-continuity)and we generalized these concepts in bi -supra topological space .At last many important theorems in strongly faintly M-θ-i-continuous functions are investigated. And study the relationships among these functions and other forms are discussed.

Key Words and Phrases: bi-supra topological space ,strongly μ-θ- i-continuity, strongly faintly μ- i -continuity.

1. Introduction

Nasef and Noiri [1] introduce three classes of strong forms of faintly continuity namely: strongly faint semicontinuity, strongly faint precontinuity and strongly faint β-continuity. Recently Nasef [2] defined strong forms of faint continuity under the terminologies strongly faint α-continuity and strongly faint γ-continuity. In this paper using μ-θ- i -open sets, strongly faintly μ-θ- i -continuity is introduced and studied in bi-supra topological spaces (Let X be non-empty set , let $\mathcal{S}o(X)$ be the set of all semi open subset of X non-empty set , let $\mathcal{S}o(X)$ be the set of all semi open subset of X (for short $\mathcal{ST}$) and let $\mathcal{P}o(X)$ be the set of all pre open subset of X(for short $\mathcal{PT}$), then we say that (X,$\mathcal{ST}$,$\mathcal{PT}$) is a bi-supra topological space, when each of (X,$\mathcal{ST}$) and (X,$\mathcal{PT}$) is a supra topological space)[3] Moreover, basic properties and preservation theorems of strongly faintly μ-θ- i - continuous functions are investigated and relationships between strongly faintly μ-θ- i - continuous functions and graphs are investigated.

2. Preliminaries.

Throughout this paper (X ,$\mathcal{T}_x$) and (Y, $\mathcal{T}_y$) (Simply, X and Y) represent topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure of subset A of X, the interior of A and the complement of A is denoted by cl(A), int(A) and A^c or X\A respectively [3] A subset A of a space X is said to be regular open if it is the interior of its closure, i.e, A = int(cl(A)). The complement of a regular- open set is referred to as a regular –closed set. A union of regular-open sets is called δ-open [4] The complement of a δ-open set is referred to as a δ-closed set. A subset A of a space(X, $\mathcal{T}_x$)is called θ-open set [5] if there exists an open set U containing x such that U⊆cl(U) ⊆ A. The set of all θ-interior points of A is said to be the θ-interior set and denoted by θ-int (A). A subset A of X is called θ-open if A = θ- int (A) The family of all θ- open sets in bi-supra topological space (X, , $\mathcal{T}_x$) forms a supra topology $\mathcal{T}_x$ on X. A subset A of a space (X, $\mathcal{T}_x$) is called semi-open [6] (resp, α-open [7], pre-open [8] β-open [9], γ- open [10]) if A⊆cl(int(A)(resp , A ⊆ int(cl(int(A)) A ⊆ int(cl(A)) A ⊆cl(int(cl(A)), cl(int(A))) . The complement of semi-open (resp. α-open , pre-open ,β-open , γ-open) set is called semi-closed (resp. α-closed , pre- closed ,β- closed , γ-closed [11]).

A of a space (X, $\mathcal{T}_x$) is called semi- θ-open (resp. α-θ -open [12], pre- θ -open ,β- θ -open ,γ- θ –open[12])if and only if for each x ∈ A there exists G∈ semi-open (resp , α-open ,pre-open , β-open , γ-open) such that semi-CL(G) ⊆A (resp. α-CL(G) ⊆A, pre- CL(G) ⊆A, β-CL(G) ⊆A, γ- CL(G) ⊆A).

The union of all θ-open (resp,semi- θ-open, δ-open,) sets contained in A is called the θ-interior (resp,semi- θ-interior [2], δ-interior) of A and it is denoted by θ- int(A) (resp, semi- θ-int(A), δ-int (A)). The intersection of all θ-closed (resp.,semi- θ- closed, δ- closed,) sets containing A is called the θ- closure (resp.,semi- θ- closure, δ- closure [13]) of A and it is denoted by θ- cl(A) (resp, semi- θ-cl(A), δ-cl(A)). We recall the following definitions and results, which are useful in the sequel

Definition 2.1[14]: Let (X ,$\mathcal{T}_x$) be a topological space. Then a subset A of X is said to be:
(i) an M-open set, if A ⊆ cl(int_θ (A)) [int(cl_δ(A)),
(ii) an M-closed set if int(cl_θ(A)) \ cl(int_δ(A)) ⊆A.

Definition 2.2[14]: Let (X ,$\mathcal{T}_x$) be a topological space and A ⊆ X.Then:
(i) the M-interior of A is the union of all M-open sets contained in A and is denoted by M-int(A),
(ii) the M-closure of A is the intersection of all M-closed sets containing A and is denoted by M-cl(A)

Definition 2.3[15]: A function f : (X, $\mathcal{T}_x$)→ (Y , $\mathcal{T}_y$) is said to be:
(i) M-continuous [14] if f^{-1} (U) ∈ M-open in X , for each U∈ $\mathcal{T}_y$
(ii) pre-M-open [16] if, f(U) ∈ M-open in Y , for each U∈ M- open in X.
(iii) pre-M-closed [11] if, f(U) ∈ M -closed in Y , for each U∈ M- closed in X.

Definition 2.4: A function f : (X,$\mathcal{T}_x$)→ (Y , $\mathcal{T}_y$) is said to be strongly faintly continuous [17] (resp. strongly faintly semicontinuous, strongly faintly pre continuous [18], strongly faintly β-continuous [18], strongly faintly α-continuous [1], strongly faintly γ- continuous[2]) i f for each x ∈ X and each open (resp.semi- open, pre -open, β-open, α-open, γ-open) set V of Y containing f(x), there exists a θ-open set U of X containing x such that f(U) ⊆ V.

3 . A new type of bi-supra topological *space*

Definitions 3.1: Let (X, $\mathcal{ST}_x$, $\mathcal{PT}_x$)be a bi-supra topological space, and let G be a subset of X. Then G is said to be i- open set if G=(AUB) U∅ where A∈$\mathcal{ST}$ and B∈$\mathcal{PT}$. The Complement of i- open set is called i- closed set.

Definitions 3.2: A subset A of a space (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$) is called θ-i-open set if there exists an i- open set U containing x such that U ⊆i- cl(U) ⊆ A.

Definitions 3.3: The set of all θ-i-interior points of A is said to be the θ-i-interior set and denoted by θ-i-int (A). A subset A of X is called θ-i-open if A = θ-i- int (A).

Definitions 3.4: A of a space (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$)is called semi- θ-i-open (resp. α- θ -i-open , pre- θ -i-open , β- θ -i-open , γ- θ -i-open)if and only if for each x ∈ A there exists G in semi-i-open (resp. , α-i-open , pre-i-open ,β-i-open , γ-i-open) such that semi-i-CL(G) ⊆A (resp. α-i CL(G) ⊆A -, pre-i- CL(G) ⊆A , β-i CL(G) ⊆A , γ-i- CL(G) ⊆A).

Definitions 3.5: A subset A of a space (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$)is called M-i-open if A ⊆ i- cl(θ-i int(A)) ∪ i- int(δ –i-cl(A)). The complement of semi-i-open (resp. , α-i-open , pre-i-open ,β-i-open , γ-i-open ,semi- θ-i-open, θ-i-open, M-i-open) set is called semi-i-closed (resp., α-i-closed , pre-i- closed ,β-i-closed , γ-i- closed ,semi- θ-i- closed, θ-i- closed, M-i- closed) . The union of all M-i-open (resp. θ-i-open,semi- θ-i-open, δ-i-open,) sets contained in A is called the M-i-interior (resp. θ-i-interior ,semi- θ- i-interior , δ-i-interior) of A and it is denoted by M-i-int(A) (resp. . θ-i- int(A), semi. θ-i-int(A), δ-i-int (A)). The intersection of all M-i-closed (resp. . θ-i-closed ,semi- θ-i- closed, δ-i- closed,) sets containing A is called the M-i-closure (resp. θ-i- closure ,semi- θ-i- closure, δ-i- closure) of A and it is denoted by M-i-cl(A) (resp. θ-i- cl(A), semi. θ-i-cl(A), δ-i-cl(A)).

Definition 3.6: A subset A of a space (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$) is called M-θ-i- open if and only if for each x ∈ A there exists G in M-i-open in X such that M-i-cl(G) ⊆A.

Also , clearly every M-θ-i- open (resp. M-θ-i- closed) set is M-i- open (rsp. M-i- closed) set. And every θ-i- open (resp. θ-i- closed) set is M-i- open (rsp. M-i-closed).

Example 3.1. Let X = {a, b, c} and $\mathcal{T}_x$ = { ∅, {a}, {a, c}, {b, c}, X}

{ ∅,{b, c}, {b}, {a}, X} , $\mathcal{ST}_x$= { ∅, {a}, {a, c}, {b, c}, X}= $\mathcal{T}_x{}^c$

$\mathcal{PT}_x$ = { ∅, {a},{c}, {a, b} {a, c}, {b, c}, X}

i-open in X = { ∅, {a},{c}, {a, b} {a, c}, {b, c}, X}, i-closed in X = { ∅, {b, c} {a, b},{c},{b} ,{a}, X}

= { ∅, {a, b}, {b, c}, X} in X θ-i- open

= { ∅, {c}, {a}, X} in X θ-i-closed

Regular –i-open in X= { ∅, {a},{c}, {a, b} , {b, c}, X}

δ-i-open in X= { ∅, {a},{c}, {a, b} {a, c}, {b, c}, X}

δ-i-closed in X= { ∅,{b, c} {a, b},{b},{c} ,{a} , X}

M-i-open in X= { ∅, {a},{c}, {a, b} ,{a,c}, {b, c}, X}

M-i-closed in X = { ∅, {b, c} {a, b},{c},{b},{a} ,X}

M-θ-i- open in X= { ∅, {a, b} {b, c} ,X}

Then {a},{c} are M-i-open set but not M-θ-i- open set in X .

Definition 3.7: The M-θ-i-closure of a subset A in bi-supra topological space (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$) is denoted by M-θ-i-cl(A) and is defined to be the set of all points x of X such that for each G in M-θ-i- open in X, M-θ-i-cl(G) ∩ A ≠ ∅.

Definition 3.8: A subset A in bi-supra topological space (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$) is said to be M-θ-i-closed if M-θ-i-cl(A) = A. The complement of a M-θ-i- closed set is called a M-θ-i -open set.

We recall the following definitions and results, which are useful in the sequel.

Definition 3.9: A function f : (X, $\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y, $\mathcal{ST}_y$, $\mathcal{PT}_y$) is said to be strongly faintly $-\theta-$i-continuous (resp. . strongly faintly semi-$\theta - i -$ continuous, strongly faintly pre θ-i-continuous, strongly faintly β- θ-i--continuous , strongly faintly α- θ-i-continuous , strongly faintly γ-θ-i-continuous) i f for each x ∈ X and each θ-i-open (resp.semi θ-i-open, pre θ-i-open, β- θ-i-open, α- θ-i-open, γ- θ-i-open) set V of Y containing f(x), there exists a θ-i-open set U of X containing x such that f(U) ⊆ V.

Definition 3.10: A function f : (X,$\mathcal{T}_x$)→ (Y , $\mathcal{T}_y$) (resp, f : (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y,$\mathcal{ST}_y$, $\mathcal{PT}_y$) is said to be:

(i) θ -continuous [19](resp. θ-i –continuous), if f^{-1} (V) is θ-open (resp. θ-i-open) in X for every open (resp. i- open)set V of Y.

(ii) quasi θ-continuous [20](resp. quasi θ-i-continuous) , if f^{-1}(V) is θ-open(resp. θ-i-open)set in X for every θ-open (resp. θ-i-open)set V of Y.

(iii)faintly continuous [15](resp. faintly i-continuous) if f^{-1}(V) is open(resp. i-open)set in X for every θ-open (resp. θ-i-open) set V of Y.

Definition 3.11: A function f : (X, $\mathcal{T}_x$)→ (Y , $\mathcal{T}_y$)(resp, f : (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y,$\mathcal{ST}_y$, $\mathcal{PT}_y$)) is said to be:

(i) M-continuous [14] (resp, M-i-continuous) if f^{-1} (U) ∈ M-open(resp, M-i-open) set in X , for each U∈ Y

(ii) pre-M-open [16](resp, pre-M- θ-i –open) if, f(U) ∈ M-open (resp, M- θ-i-open) set in Y , for each U∈ M-open(resp, M- θ -i-open) set in X.

(iii) pre-M-closed [16] (resp, pre-M- θ-i –closed) if, f(U) ∈ M -closed (resp, pre-M- θ-i –closed) set in Y , for each U∈ M- closed(resp, pre-M- θ-i –closed)set in X.

Lemma 3.1:For topological space (X,$\mathcal{T}_x$) (resp,bi-supra topological space (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) and A ⊆ X, then the following statements are hold:

(i) If A ⊆ Fi, Fi is an M-closed [15](resp, M-i-closed)set of X, then A ⊆ M-cl(A) ⊆ Fi,(resp.M -i-cl(A) ⊆ Fi).

(ii) If Gi ⊆ A, Gi is an M-open[14] (resp . M-i-open) set of X, then Gi ⊆ M-int(A) ⊆ A(resp. Gi ⊆ M-i-int(A) ⊆ A) .

Proposition 3.1: Let $(X,\mathcal{T}_x)$ topological space (resp. $(X,\mathcal{ST}_x,\mathcal{PT}_x)$ be bi-supra topological space) and A ⊆ X. Then . the following statements are hold:

(i) θ-F_r (A) = θ-cl (A) \ θ-int (A))[9] (resp. θ-i-F_r (A) = θ-i cl (A) \(θ-i int (A))

(ii) M-F_r (A) = M-cl(A) \ M-int(A) [13] (resp.M-i-F_r (A) = M-i-cl(A) \(M-i-int(A))

(iii) M- b(A) = A\M-int(A) [11] (resp. M-i- b(A) = A\M-i-int(A)).

The set of θ-boundary (resp. θ-i-boundary ,M-i-boundary, M-i-border) of A is denoted by θ-F_r (A) (resp. θ-i-F_r (A),M-i-F_r (A), M-i- b(A)).

4. strongly faintly M-θ-i –continuous functions.

Definition 4.1: A function f : $(X,\mathcal{ST}_x,\mathcal{PT}_x)\rightarrow(Y,\mathcal{ST}_y,\mathcal{PT}_y)$ is said to be strongly faintly M- θ-i -continuous if for each x ∈ X and each M- θ-i -open set V of Y containing f(x), there exists a θ-i-open set U of X containing x such that f(U) ⊆ V

Remark 4.1 : The implication between some types of definitions (**3.9**) and (**4.1**) are given by the following diagram.

Faintly-i-continuous ⟵ s.faintly M- θ-i-continuous

↖

↑ s.faintlyα - θ-i-continuous

↗ ↖

s.faintly pre - θ-i-continuous | s.faintly semi θ-i-continuous

↖ ↗

s.faintly γ- θ-i-continuous

↑

s.faintly β - θ-i-continuous

The proof of implication form definition directly .However, none of these implications is reversible as shown by the following examples and well- known facts.

Example 4.1 : Let X=Y = {a, b, c}

and $\mathcal{T}_x$ = { ∅, {a},{b}, {a, b}, {a, c}, X}

{ ∅,{b, c},{a,c}, {c}, {b}, X} **=** $\mathcal{T}_x^{\,c}$

$\mathcal{ST}_x$= { ∅, {a},{b}, {a, b}, {a, c}, X} , $\mathcal{PT}_x$ = { ∅, {b}, {a, b} {a, c}, X}

i-open in X = { ∅, {a},{b}, {a, b} {a, c}, X}

i-closed in X = { ∅, {b, c} {a, c},{c},{b} , X}, θ-i-open in X={ ∅, {a, c}, X}

with $\mathcal{T}_Y$ = { ∅, {a, c}, Y}

{ ∅,{b}, Y} **=** $\mathcal{T}_Y^{\,c}$

$\mathcal{ST}_Y$= { ∅, {a, c}, Y} , $\mathcal{PT}_Y$ = { ∅, {b}, {a, b} {b, c}, Y}

i-open in Y = { ∅,{b}, {a, b} {a, c},{b,c}, Y} , i-closed in Y = { ∅, {a, c} , {c},{b},{a} , Y}

semi-i-open in Y={ ∅,{b}, {a, b} {a, c},{b,c}, Y}

semi-i-closed in Y= { ∅ , {a, c} { c},{b},{a}, Y} , semi- θ-i- open in Y={ ∅ , {a, c}, Y}

γ-i-open in Y={ ∅ ,{a},{b},{a,b} {a, c},{b,c} Y}

γ-i-closed in Y={ ∅ ,{b,c},{a,c},{c } {b},{a} Y}

define the function f : $(X,\mathcal{ST}_x,\mathcal{PT}_x)\rightarrow(Y,\mathcal{ST}_y,\mathcal{PT}_y)$ by the identity function . then f is strongly faintly semi- θ-i-continuous but not strongly faintly γ - θ-i-continuous.

Example 4.2: Let R be the set of real numbers $\mathcal{T}_x$ the indiscrete topology for R and $\mathcal{T}_Y$ the discrete topology for R.

Then the identity function f : $(X,\mathcal{T}_x)\rightarrow(X,\mathcal{T}_y)$(resp, f : $(X,\mathcal{ST}_x,\mathcal{PT}_x)\rightarrow(Y,\mathcal{ST}_y,\mathcal{PT}_y)$) is strongly faintly pre-continuous but not strongly faintly γ -continuous [9] (resp, strongly faintly pre- θ-i-continuous but not strongly faintly γ - θ-i-continuous).

Example 4.3: Let $\mathcal{T}$x be the uasul topology for R and $\mathcal{T}_Y$ ={ R,∅ , [0,1]∪ (1,2) ∩ Q} where Q denotes the set of the set of rational numbers. Then the identity function .

f : $(X,\mathcal{T}_x)\rightarrow(X,\mathcal{T}_y)$(resp, f : $(R,\mathcal{ST}_x,\mathcal{PT}_x)\rightarrow(R,\mathcal{ST}_y,\mathcal{PT}_y)$) is strongly faintly γ - continuous but not strongly faintly β –continuous[20] (resp, strongly faintly γ - θ-i-continuous but not strongly faintly β - θ-i-continuous.

Example 4.4: (1):in ([1], example 3.2) showed a strongly faintly semi-continuity (resp,strongly faintly semi- θ-i-continuity) which is not a strongly faintly pre-continuity. (resp,strongly faintly pre- θ-i-continuity).

(2): using example 3.2 of [8] , this easily observed that a strongly faintly α -continuity(resp, faintly α - θ-i-continuity) but not strongly faintly M –continuity (resp, not strongly faintly M - θ-i-continuity) .

Theorem 4.1: For a function f : $(X,\mathcal{ST}_x,\mathcal{PT}_x)\rightarrow(Y,\mathcal{ST}_y,\mathcal{PT}_y)$ the following statements are equivalent:

(i) f is strongly faintly M - θ-i-continuous.

(ii) f: $(X,\mathcal{ST}_x,\mathcal{PT}_x)\rightarrow(Y,\mathcal{ST}_y,\mathcal{PT}_y)$ is faintly-i-continuous.

(iii) f^{-1} (V) is θ-i-open in X for every M - θ-i -open set V of Y (iv) f^{-1} (F) is θ-i-closed in X for every M - θ-i -closed subset F of Y.

Proof. (i)⇒(iii): Let V be an M - θ-i -open set of Y and x ∈ f^{-1} (V). Since f(x) ∈ V and f is strongly faint M - θ-i -continuous, there exists a θ-i-open set U of X containing x such that f(U) ⊂ V. It follows that x ∈ U ⊂ f^{-1} (V).

Hence f^{-1} (V) is θ-i-open in X.

(iii)⇒(i): Let x ∈ X and V be an M - θ-i -open set of Y containing f(x). By (iii), f^{-1} (V) is a θ-i-open set containing x. Take U = f^{-1} (V). Then f(U)⊆ V.

This shows that f is strongly faint M - θ-i -continuous.

(iii)⇒(iv): Let V be any M - θ-i -closed set of Y. Since Y \ V is an M - θ-i -open set, by (iii), it follows that f^{-1} (Y\V) = X\f^{-1} (V) is θ-i-open. This shows that f^{-1} (V) is θ-i-closed in X.

(iv)⇒(iii): Let V be an M - θ-i -open set of Y. Then Y\V is M - θ-i -closed in Y. By(iv), f^{-1} (Y\V) = X\f^{-1} (V) is θ-i-closed and thus f^{-1} (V) is θ-i-open.(i)⇔(ii): Clear.

Now we intuduce a very important theorem to explam the equivalence implication.

Theorem 4.2: For a function f : (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y, $\mathcal{ST}_y$, $\mathcal{PT}_y$), the following statements are equivalent:

(i) f is strongly faintly M- θ-i -continuous,

(ii) For each x∈ X and each M- θ-i -open V of f(x) in Y, there exists an θ-i -open set U of x in X such that f (U) ⊆ V,

(iii) f^{-1} (F) is θ-i -closed in X, for every M- θ-i-closed set F of Y,

(iv) θ-i -cl(f^{-1} (B)) ⊆ f^{-1} (M-θ-i cl (B)), for each B ⊆ Y,

(v) f(θ-i -cl(A)) ⊆M- θ-i cl (f(A)), for each A ⊆ X,

(vi) f^{-1} (M-θ-i- int(B)) ⊆ θ-i -int(f^{-1} (B)), for each B ⊆ Y,

(vii) θ-i –b (f^{-1} (B)) ⊆ f^{-1} (M- θ-i- b(B)) , for each B ⊆ Y,

(viii) θ-i -Fr(f^{-1} (B)) ⊆ f^{-1} (M-θ-i -Fr(B)), for each B ⊆ Y.

Proof. (i)→(ii). Let x∈ X and V ⊆ Y be A M- θ-i-open set containing f(x). Then x∈ f^{-1} (V). Hence by hypothesis, f^{-1} (V) is θ-i-open set of X containing x. We put U = f^{-1} (V).

Then x∈ U and f(U) ⊆ V.

(ii)→(iii). Let F ⊆ Y be M-θ-i-closed. Then Y\F is M-θ-i-open and x∈ f^{-1} (Y\F). Then f(x) ∈ Y\F. Hence by hypothesis, there exists an θ-i-open set U containing x such that f(U) ⊆ Y\F, this implies that, x∈ U ⊆ f^{-1} (Y\F). Therefore, f^{-1} (Y\F) = X\ f^{-1} (F) which is θ-i-open in X. Therefore, f^{-1} (F) is θ-i closed.

(iii)→(i). Let V ⊆ Y be a M-θ-i-open set. Then Y\V is M-θ-i-closed in Y. By hypothesis, f^{-1} (Y\V) = X\f^{-1} (V) is θ-i -closed and hence f^{-1} (V) is θ-i-open. Therefore, f is strongly faintly M- θ-i -continuous.

(i)→ (iv). Since B ⊆ M-θ-i cl(B) ⊆ Y which is a M-θ-i-closed set. Then by hypothesis, f^{-1} (clθ(B)) is θ-i -closed in X . Hence by Lemma 2.1, θ-i-cl(f^{-1} (B)) ⊆ f -1(M-θ-i cl(B)) for each B ⊆ Y.

(iv) →(v). Let A ⊆ X. Then f(A) ⊆Y, hence by hypothesis, θ-i -cl(A) ⊆ θ-i -cl(f^{-1} (f(A))) ⊆ f^{-1} (M-θ-i cl(f(A))). Therefore, f(θ-i -cl(A)) ⊆ f f^{-1} (M-θ-i -cl(f(A))) ⊆M- θ-i- cl(f(A)), (v) →(i). Let V ⊆ Y be a M-θ-i-closed set. Then, f^{-1} (V) ⊆ X . Hence, by hypothesis, f(θ-i -cl(f^{-1}V))) ⊆ M- θ-i -cl(f(f^{-1} (V))) ⊆ M- θ-i -cl(V) = V. Thus θ-i -cl(f^{-1} (V)) ⊆ f^{-1} (V) and hence f^{-1} (V) ∈ θ-i –closed in X .Hence, f is strongly faintly M- θ-i -continuous,

(i) → (vi). SinceM -θ-i - int(B) ⊆ B ⊆ Y is M- θ-i-open. Then by hypothesis, f^{-1} (M- θ-i -int(B)) is an θ-i -open set in X. Hence, by Lemma 2.1, f^{-1} (M- θ-i -int(B)) ⊆ θ-i -int(f^{-1} (B)), for each B ⊆ Y.

(vi) → (i). Let V ⊆ Y be a M-θ-i-open set. Then by assumption, f^{-1}(V) = f^{-1} (M- θ-i -int(V)) ⊆ θ-i -int(f^{-1} (V)) . Hence, f^{-1} (V) is θ-i -open in X. Therefore, f is strongly faintly M- θ-i continuous.-

(vi) → (vii). Let V ⊆ Y. Then by hypothesis, f^{-1} (M- θ-i -int(V)) ⊆ θ-i -int(f^{-1} (V)) and so f^{-1} (V) \ θ-i -int(f^{-1} (V)) ⊆ f^{-1} (V) \ f^{-1} (M- θ-i -int(V)) = f^{-1} (V\M- θ-i - int(V)). By Proposition (3.1), θ-i -b(f^{-1} (V)) ⊆ f^{-1} (M- θ-i-b(V)).

(vii) → (vi). Let V ⊆ Y. Then by hypothesis, f^{-1} (V) \ θ-i -int(f (V)) ⊆ f^{-1} (V) \ f^{-1} (M- θ-i int(V)). Therefore, f^{-1} (M- θ-i -int(V)) ⊆ θ-i -int(f^{-1} (V)).

(vi) → (viii). Let B ⊆ Y. Then by (vi), f^{-1} (M- θ-i –int(B)) ⊆ θ-i -int(f^{-1} (B)) . Hence by (iv), θ-i -cl(f^{-1} (B)) \ θ-i -int(f^{-1}-(B)) ⊆ f^{-1} (M- θ-i -cl(B)) \ f^{-1} (M- θ-i -int(B)) . So, by Proposition (3.1), θ-i -Fr(f^{-1} (B)) ⊆ f^{-1} (M- θ-i -Fr(B)), for each B ⊆ Y.

(viii)→(vi). Let B ⊆ Y. Then by Proposition(3.1), θ-i -Fr(f^{-1} (B)) = θ-i -cl(f^{-1} (B)) \ θ-i -int(f^{-1} (B)) ⊆ f^{-1} (M- θ-i -cl(B)) \ f^{-1} (M- θ-i -int(B)) this implies that f^{-1} (M- θ-i -int(B)) ⊆ θ-i -int(f^{-1} (B)), for each B ⊆ Y.

Definition 4.2: A function f : (X, $\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y, $\mathcal{ST}_y$, $\mathcal{PT}_y$) is said to be strongly M- θ-i-continuous , if for each x∈ X and each i-open set V of Y containing f(x) , there exists U∈ M-i- open in X such that f(M-i -cl(U))⊆ V.

Proposition 4.1: If a function f : (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y,$\mathcal{ST}_y$, $\mathcal{PT}_y$) is strongly faintly M- θ-i-continuous then f is strongly M- θ-i-continuous .

Remark 4.2: the converse of the above proposition is not true as shown by the following example.

Example 4.5: Let X=Y = {a, b, c} with topologies

$\mathcal{T}$x = { ∅, {a},{b}, {a, b}, {a, c}, X}

{ ∅,{b, c},{a,c}, {c}, {b}, X} **=** $\mathcal{T}_x^c$

$\mathcal{ST}_x$= { ∅, {a},{b}, {a, b}, {a, c}, X} , $\mathcal{PT}_x$ = { ∅, {b}, {a, b} {a, c}, X}

i-open in X = { ∅, {a},{b}, {a, b} {a, c}, X}i-closed in X = { ∅, {b, c} {a, c},{c} ,{b} X}

θ-i- open in X={ ∅,{a, c},X} , θ-i- closed in X={ ∅,{b},X}

regular –i-open in X = { ∅, {b},{c}, {a, c}, X}

δ-i-open in X= { ∅, {b},{c}, {a, c},{b,c}, X}, δ-i-closed in X= { ∅,{a, c},{a,b},{a},{b}, X}

M-i- open in X= { ∅, {b},{c}, {a, b} {a, c},{b,c}, X}, M-i- closed in X= { ∅, {a},{b}, {a, b} {a, c},{b,c}, X},

And let $\mathcal{T}_Y$ = { ∅, {a, c}, Y},

{ ∅,{b}, Y} **=** $\mathcal{T}_y^c$

$\mathcal{ST}_Y$= { ∅, {a, c},Y}, $\mathcal{PT}_Y$ = { ∅, {b}, {a,b} {b, c}, Y}

i-open in Y = { ∅,{b}, {a, b} {a, c},{b,c},Y }, i-closed in Y = { ∅,{a, c} { c},{b},{a},Y },

θ-i- open in Y={ ∅,{a, c},Y}, θ-i- closed in Y={ ∅,{b},Y},

regular –i-open in Y = { ∅,{b}, {a, c}, Y}

δ-i-open in Y= { ∅, {b}, {a, c}, Y}, δ-i-closed in Y= { ∅, {b}, {a, c}, Y},

M-i- open in Y= { ∅,{b}, {a, b} {a, c}, Y}

M-i- closed in Y= { ∅,{a},{b},{c}, {a, b} {a, c}, {b,c},Y}

M- θ-i- open in Y={ ∅, {a,b},{a, c},Y}

define the function f : (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y,$\mathcal{ST}_y$, $\mathcal{PT}_y$) by the identity function . then f is strongly M- θ-i-continuous but not strongly faintly M- θ-i-continuous.

Proposition 4.1: If f : (X, $\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y,$\mathcal{ST}_y$, $\mathcal{PT}_y$) is strongly faintly M- θ-i-continuous then:
(i) f is quasi θ-i-continuous
(ii) f is faintly i- continuous.

Proof.(i): Let x∈X and V ⊆ Y be M- θ-i-open containing f(x). then there exist an θ-i-open set U such that f(U) ⊆ V .since every M- θ-i-open is θ-i-open set if f^{-1} (V) ∈ θ-i-open in X for every V∈ θ-i-open in Y .then f is quasi θ-i-continuous .
(ii) similar (i)

Definition 4.3: A function f : (X, $\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y,$\mathcal{ST}_y$, $\mathcal{PT}_y$) is called weakly-M- θ-i -continuous if, for each x∈X and each i-open set V of Y containing f(x), there exists M- θ-i –open in X such that f(U) ⊆ i-cl(V).

Theorem 4.4: the following statements are hold for function
f : (X, $\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y, $\mathcal{ST}_y$, $\mathcal{PT}_y$) and g: (Y,$\mathcal{ST}_y$, $\mathcal{PT}_y$)→ (Z,$\mathcal{ST}_z$,$\mathcal{PT}_z$)
(i) If, f is quasi θ-i-continuous and g is strongly faintly M- θ-i-continuous, then g o f is strongly faintly M- θ-i-continuous,
(ii) If, f is strongly faintly M- θ-i-continuous and g is weakly-M θ-i -continuous, then g o f is θ-i –continuous.

Proof. (i) Let V ⊆ Z be M- θ-i-open set and g be strongly faintly M- θ-i-continuous,, then g^{-1}(V) ∈ θ-i-open in Y. But f is quasi θ-i-continuous, then $(gof)^{-1}$(V) ∈ θ-i-open in Y .Hence, g o f is strongly faintly M- θ-i-continuous.
(ii) similar (i).

Theorem 4.5:.For two function f :(X,$\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y, $\mathcal{ST}_y$, $\mathcal{PT}_y$) and g: (Y, $\mathcal{ST}_y$, $\mathcal{PT}_y$)→ (Z,$\mathcal{ST}_z$,$\mathcal{PT}_z$) the following properties are hold:
(i) If, g is a surjective pre-M- θ-i -open and g o f is strongly faintly M- θ-i-continuous. then f is strongly faintly M- θ-i-continuous.
(ii) If, g is a surjective pre-M- θ-i -closed and g o f is strongly faintly M- θ-i-continuous. then f is strongly faintly M- θ-i-continuous.

Proof. (i) Let V⊆ Z be a M- θ-i -open set. Since, g o f is strongly faintly M- θ-i-continuous., then $(gof)^{-1}$ (V) is θ-i –open in X . But, g is surjective pre-M- θ –i-open, then g^{-1}(V) is M- θ-i -open set in Y. Therefore ,f is faintly M- θ-i-continuous
(ii) Obvious

Definition 4.4: A function f :(X, $\mathcal{ST}_x$, $\mathcal{PT}_x$)→ (Y,$\mathcal{ST}_y$, $\mathcal{PT}_y$) is called :
(i) M-θ-i-open if f(V) ∈ θ-i-open in Y for each V ∈ M- θ-i-open in X,
(ii) M-θ-i-closed if f(V) ∈ θ-i-open in Y for each V ∈ M- θ-i-closed in X.

References

[1] A.A. Nase, T. Noiri, Strong forms of faint continuity, Mem. Fac. Sci. Kochi Univ. Ser. A. Math, 19, 21-28, (1998)
[2] A.A. Nase , Recent progress in the theory of faint continuity, Mathematical and Computer Modelling, 49, 536-541 , (2009),
[3] Firas N. AL-Zandi , A new type of bi-supra topological spaces , Thesis in, A.D , 2014
[4] M.H. Stone, Application of the theory of Boolean rings to general topology, Tams. , 41, 375 – 381, (1937)
[5] P.E. Long, L.L. Herrington, The τθ-topology and faintly continuous functions, Kyungpook Math. J. 22, 7–14, (1982)
[6] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 68 ,44-46, (1961)
[7] O. Njåstad, On some classes of nearly open sets, Pacfic J. Math. 15, 961–970, (1965)
[8] A. S. Mashhour, M. E. Abd-El Monsef, and S. N. El-Deeb, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, 53, 47–53, (1982)
[9] M. E. Abd El-Monsef, El-Deeb S. N. and R. A. Mahmoud, β-Open sets and **β**-continuous mappings, Bul l. Fac. Sci. Assiut Univ., 12, 77–90 , (1983)
[10] A.A. El-Atik, A study on some types of mappings on topological spaces, M. Sc. Thesis Tanta Univ., Egypt (1997).
[11] A.I. EL-Maghrabi and M.A. AL-Juhani, M-open sets in topological spaces, Pioneer J. Math. Sciences, 4, no. 2, 213-230 , (201 1)
[12] B. Roy and T. Noiri, Unification on strongly θ-continuous functions, Analele Universit˘at¸ ii de Vest, Timi¸ soara, Seria Matematic˘a – Informatic˘a LI, 2, 115– 123, (2013)
[13] E. Ekici, On e-open sets, DP*-sets and DPE*-sets and decompositions of continuity , Arabian J. Sci. Eng., 33, no. 2A, 269 – 282, (2008)
[14] A.I. EL-Maghrabi and M.A. AL-Juhani, Further properties on M-continuity, Proc. Math. Soc. Egypt, 22, 63- 69, (2014)
[15] A.I.EL-Maghrabi and M.A.AL-Juhani , New weak forms of faint continuity, IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 1 Issue 4, 6 ,285 – 295 , (2014)
[16] A.I. EL-Maghrabi and M.A. AL-Juhani, New types of functions by M-open sets, Taibah Univ. J. Sci., 7, 137-145 , (2013)
[17] M. Caldas, A strong form of strong θ – continuity, General Mathematics Vol. 20, No. 4 ,27–42 , (2012).
[18] A.A. Nase, Strongly β-irresolute functions, J.Natur. Sci. Math., 36, 199-206, (1996).
[19] S. Fomin, Extensions of topological spaces, Ann. Math., 44, 471- 480 , (1943)
[20] S. Jafari, Some properties of quasi θ-continuous functions, Far East J. Math. Soc., 6, 689-696 , (1998).

حول الدوال القوية الضعيفة من النمط i- θ -M في الفضاء ثنائي التبولوجي الفوقي

طه حميد جاسم[1] ، ريم عمران رشيد[2]

[1]قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

[2]قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة تكريت ، تكريت ، العراق

الملخص

في هذا البحث قدمنا تعريفا جديدا أسمينا i-open ومن خلال هذا التعريف قدمنا صف من المفاهيم التبولوجيه (مجموعه مفتوحة من النمط **i- θ -M** , مجموعه مغلقة من النمط **i- θ -M** , الدوال القوية الضعيفة من النمط **θ -M** , الدوال القوية من النمط **θ -M**) وعممنا هذه المفاهيم في الفضاء ثنائي التبولوجي الفوقي وأجراء عدة مبرهنات مهمة في هذا الموضوع قد برهنت ودرسنا العلاقات بين تلك الدوال وافترضنا أشكال أخرى.