ON VIASMAN-GRAY MANIFOLD WITH GENERALIZED CONHARMONIC CURVATURE TENSOR

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ABSTRACT

The current study deals with the generalized conhormonic curvature tensor of Vaisman-Gray manifold. The aim of this paper to calculate the components of generalized Ricci tensor and generalized Riemannian tensor of VG-manifold in the adjoint G-structure space to find Generalized conharmomic Curvature tensor of VG-manifold, one of the almost hermitian manifold structures is donated by W₁ ⊕ W₄, where W₁ and W₄ respectively denoted to the nearly kahler manifold and locally conformal kahler manifold have been studied.

1. Introduction

In (1957) Y. Ishi [6] studies conharmomic transformation which is a conformal transformation. One of the representative work of differential geometry is an almost Hermitian structure. In particular, the problem of classification of this structure. Gray and Hervella [5] found the action of the unitary group U(n) on the space of all tensors of type (3,0) decomposed this space into sixteen classes. In 1993, Banaru [3] succeeded in re-classifying the sixteen classes of almost Herm manifold by using the structure and virtual tensors, which were named Krichikhon's tensors [2]. Among the sixteen classes of almost hermitian manifold, there are eight which is invariants under the conformal transformation manifold.

This research tackles the almost kahler and nearly kahler manifold so it found their important geometrical properties. In 2006 Krolikowski proved that there is an 4-dimensional almost kahler manifold of locally conformally flat with a metric of a special form [8]. In 2010, Lafta studied conharmomic curvature tensor of classes almost kahler and nearly kahler manifolds [9]. In 2016 [13] Habeed M. Abood and Yasir A. Abdulameer where studied the flatness of conharmomic curvature tensor of VG-manifold in using the method of an adjoint G-structure space and I have proved that the compounds of conharmomic curvature tensor of VG-manifold and Riemannian curvature tensor and Ricci tensor of VG-manifold in the adjoint G-structure space. In 2018 [1] Ali A. Shihab and Dhabia’a M. Ali where studied generalized conharmomic curvature tensor of nearly Kahler manifold. In the study also concentrates generalized conharmomic curvature tensor of Vaisman-Gray manifold.

2. Preliminaries

Let M be a smooth manifold of dimension 2n, C∞(M) is algebra of smooth function on M; X(M) is the module of smooth vector fields on manifold of M ; g = <, > is Riemannian metrics; V is Riemannian connection of the metrics g on M; d is the operator of exterior differentiation. In the further all manifold tensor field, etc. objects are assumed smooth a classes C∞(M).

We concentrate our attention on generalized conharmomic tensor of Vaisman-Gray manifold, where Vaisman Gray manifold is considered as one of the most important classes of almost hermitian manifold which is denoted by W₁ ⊕ W₄ and represents a generalization of the W₁ and W₄ classes. The space W₄ is called nearly Kähler manifold (NK -manifold) and W₄ is called locally conformal Kähler manifold (LCK-manifold).

Definition 1 [5]
A Vaisman-Gray structure is an $G$-structure $[J, g = \langle \ldots \rangle]$ such that:
\[ \nabla_X(F) \times (Y, Y) = \frac{-1}{2(n-1)} \left[ \langle X, Y \rangle > \delta F(X) - \langle \ldots \rangle \right] \]
where $F$ is the Riemann connection of $g$.

By definition, the components of the Riemannian curvature tensor of $\operatorname{VG}$-manifold are given by the following forms:

1) $R_{ab} = \frac{\Gamma_{a}^{c}}{2} a_{b c} + \frac{\alpha_{a}}{2} b_{b c} + A_{a}^{c} a_{b c} + \frac{n-1}{2} \left( a^{a} \alpha_{b} - a^{b} \alpha_{a} \right) - \frac{1}{2} a^{a} \delta_{b}^{c} \delta_{b}^{c} \left( n - 2 \right) a^{a} b$

**Definition 6** [6]

Let $(M, J, g)$ be a Vaisman-Gray manifold, the Conharmonic curvature tensor of $\operatorname{AH}$-manifold $M$ of type $(4,0)$ which is defined as the following form:

- $T_{ijkl} = R_{ijkl} - \frac{1}{2(n-1)} \left[ \langle \ldots \rangle \delta_{k}^{l} g_{jk} - \langle \ldots \rangle \delta_{l}^{j} g_{ik} + \langle \ldots \rangle \delta_{k}^{i} g_{jl} - \langle \ldots \rangle \delta_{j}^{i} g_{kl} \right]$

**Remark 7** [2]

From the Banaruces classification of $\operatorname{AH}$-manifold, the class $\operatorname{VG}$-manifold satisfies the following conditions:

- $R_{abc} = -B_{bac}, B_{ab} = a_{b c} \delta_{a}^{b}$

**Definition 8** [4]

A generalized Riemannian curvature tensor on $\operatorname{AH}$-manifold $M$ is called a tensor of kind $(4,0)$ whose is defined as the following format:

\[ \langle HR \rangle(X, Y, Z, W) = \frac{1}{16} \left[ 3[R(X, Y, Z, W)] + R(X, J, Y, Z, W) + R(X, J, Y, Z, W) - R(X, J, Z, W, Y) \right] \]

**Definition 9**

A generalized Conharmonic curvature tensor (GT-curvature) tensor of Vaisman-Gray manifold $(\operatorname{VG}$-manifold) $M$ of type $(4,0)$ which is defined as the following form:

\[ \langle GT \rangle(X, Y, Z, W) = \frac{1}{16} \left[ 3[R(X, Y, Z, W)] + R(X, J, Y, Z, W) + R(X, J, Y, Z, W) - R(X, J, Z, W, Y) \right] \]

Consider this equation in the adjoin $\operatorname{G}$-structure space we get:

\[ G_{\alpha \beta \gamma} = \frac{1}{16} \left[ 3 \langle T_{\alpha \beta \gamma} \rangle + T_{\alpha \beta \gamma} + T_{\alpha \beta \gamma} + T_{\alpha \beta \gamma} - T_{\alpha \beta \gamma} - T_{\alpha \beta \gamma} - T_{\alpha \beta \gamma} - T_{\alpha \beta \gamma} - T_{\alpha \beta \gamma} - T_{\alpha \beta \gamma} \right] \]
Theorem 10
The components of the generalized Riemannian curvature tensor of VG-manifold in the adjoint G-structure space are given as the following forms:

1) \( G_{abcd} = \frac{1}{4} \left\{-3A_{bd}^{ac} - 3B_{bd}^{ac} + 3B_{ab}^{cd} + \frac{1}{2} A_{[b}^{a,c]} \right\} \)

2) \( G_{abcd} = \left\{ A_{bd}^{ac} - B_{ab}^{cd} + \frac{1}{2} A_{[b}^{a,c]} \right\} \)

Proof

By using Theorem (3) and definition (8), we calculate the components of generalized Riemannian tensor as follows:

1) For \( i = a, b, c, k = c \) and \( l = d \),

\[ G_{abcd} = \frac{1}{4} \left\{ 3T_{abcd} - T_{abc} - T_{ab}^{cd} + T_{ab}^{cd} - T_{ab}^{cd} - T_{abc} - T_{abc} - T_{abc} - T_{abc} \right\}. \]

2) For \( i = a, j = b, k = c \) and \( l = d \),

\[ G_{abcd} = \frac{1}{4} \left\{ 3T_{abcd} + T_{abc} - T_{ab}^{cd} - T_{ab}^{cd} + T_{abc} + T_{abc} + T_{abc} - T_{abc} - T_{abc} \right\}. \]

3) For \( i = a, j = b, k = c \) and \( l = d \),

\[ G_{abcd} = \frac{1}{4} \left\{ 3T_{abcd} + T_{abc} - T_{ab}^{cd} - T_{ab}^{cd} - T_{abc} - T_{abc} + T_{abc} + T_{abc} \right\}. \]

4) For \( i = a, j = b, k = c \) and \( l = d \),

\[ G_{abcd} = \frac{1}{4} \left\{ 3T_{abcd} - T_{abc} + T_{abc} - T_{abc} - T_{abc} - T_{abc} + T_{abc} + T_{abc} \right\}. \]

5) For \( i = a, j = b, k = c \) and \( l = d \),

\[ G_{abcd} = \frac{1}{4} \left\{ 3T_{abcd} - T_{abc} + T_{abc} - T_{abc} + T_{abc} - T_{abc} + T_{abc} + T_{abc} \right\}. \]

6) For \( i = a, j = b, k = c \) and \( l = d \),

\[ G_{abcd} = \frac{1}{4} \left\{ 3T_{abcd} + T_{abc} - T_{ab}^{cd} - T_{ab}^{cd} + T_{abc} + T_{abc} + T_{abc} - T_{abc} - T_{abc} \right\}. \]

7) For \( i = a, j = b, k = c \) and \( l = d \),

\[ G_{abcd} = \frac{1}{4} \left\{ 3T_{abcd} - T_{abc} + T_{abc} - T_{abc} + T_{abc} + T_{abc} + T_{abc} + T_{abc} \right\}. \]

Definition 11 [11]

A tensor of type \((2, 0)\) which is defined as \( r(GT)_{ij} = (GT)^{ij}_{\text{ik}} \) is called a generalized Ricci tensor.

Theorem 12

The components of generalized Ricci tensor of VG-manifold in the adjoint G-structure space are given as the following form:

\[ r(GT)_{ab} = (GT)_{ca} - AB^{ac} + B_{ab}^{cd} + \frac{1}{2} A_{[b}^{a,c]} \]

Proof

By using Theorem (10), we can get the components of generalized Ricci tensor as follows:

1) For \( i = a, j = b \) \( r(GT)_{ab} = (GT)^{ab}_{\text{bbk}} \)

2) For \( i = a, j = b \) \( r(GT)_{ab} = (GT)_{\text{abk}} + (GT)^{\text{ab}}_{\text{abc}} + (GT)_{\text{abc}} = (GT)_{\text{ca}b} = 0 \)
3) For i = â, j = b
\[ r(GT)_{âb} = (GT)_{âb}^k = (GT)_{âbc} + (GT)_{âbc}^c = (GT)_{âbc} + cB_{âb}h + \frac{1}{2} \alpha_{[sh]} \]

Then

\[ GT_{âbcd} = 0 - \frac{1}{2(1-n)} \left( \delta^e_{â} \right) \]

\[ GT_{âbcd} = 0 - \frac{1}{2(1-n)} \left( \delta^e_{â} \right) \]

Theorem 13
The components from the generalized conharmonic curvature of V'G-manifold in the adjunct
G-structure are given as follows:

\[ GT_{ijkl} = R_{ijkl} - \frac{1}{2(1-n)} B_{ijklr}(GT)_{jl} + B_{ijklr}(GT)_{il} - B_{ijklr}(GT)_{il} \]

Then

\[ GT_{âbcd} = 0 - \frac{1}{2(1-n)} \left( \delta^e_{â} \right) \]

\[ GT_{âbcd} = 0 - \frac{1}{2(1-n)} \left( \delta^e_{â} \right) \]

Proof:
1) For i = â, j = b, k = c and l = d
\[ GT_{âbcd} = R_{âbcd} - \frac{1}{2(1-n)} B_{âcr}(GT)_{bd} + B_{âcr}(GT)_{bc} - B_{âcr}(GT)_{ad} \]

\[ \text{GT}_{âbcd} = 0 - \frac{1}{2(1-n)} \left( \left( \delta^e_{â} \right) \right) = 0 \]

2) For i = â, j = b, k = c and l = d
\[ GT_{âbcd} = R_{âbcd} - \frac{1}{2(1-n)} B_{âcr}(GT)_{bd} + B_{âcr}(GT)_{bc} - B_{âcr}(GT)_{ad} \]

\[ \text{GT}_{âbcd} = 0 - \frac{1}{2(1-n)} \left( \left( \delta^e_{â} \right) \right) = 0 \]

3) For i = â, j = b, k = c and l = d
\[ GT_{âbcd} = R_{âbcd} - \frac{1}{2(1-n)} B_{âcr}(GT)_{bd} + B_{âcr}(GT)_{bc} - B_{âcr}(GT)_{ad} \]

\[ \text{GT}_{âbcd} = 0 - \frac{1}{2(1-n)} \left( \left( \delta^e_{â} \right) \right) = 0 \]

4) For i = â, j = b, k = c and l = d
\[ GT_{âbcd} = R_{âbcd} - \frac{1}{2(1-n)} B_{âcr}(GT)_{bd} + B_{âcr}(GT)_{bc} - B_{âcr}(GT)_{ad} \]

\[ \text{GT}_{âbcd} = 0 - \frac{1}{2(1-n)} \left( \left( \delta^e_{â} \right) \right) = 0 \]
Conclusion
The components of the generalized Riemannian curvature tensor of Viasman-Grey manifold, the components of generalized Ricci tensor of Viasman-Grey manifold, and the components of the generalized conharmonic curvature of Viasman-Grey manifold.

References

ملخص
تهنئ الانحناء الكونهورمني المعمم لمنطوي فايسمان- كراي

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